

String Art

Student Worksheet

7 8 9 10 11 12



Introduction

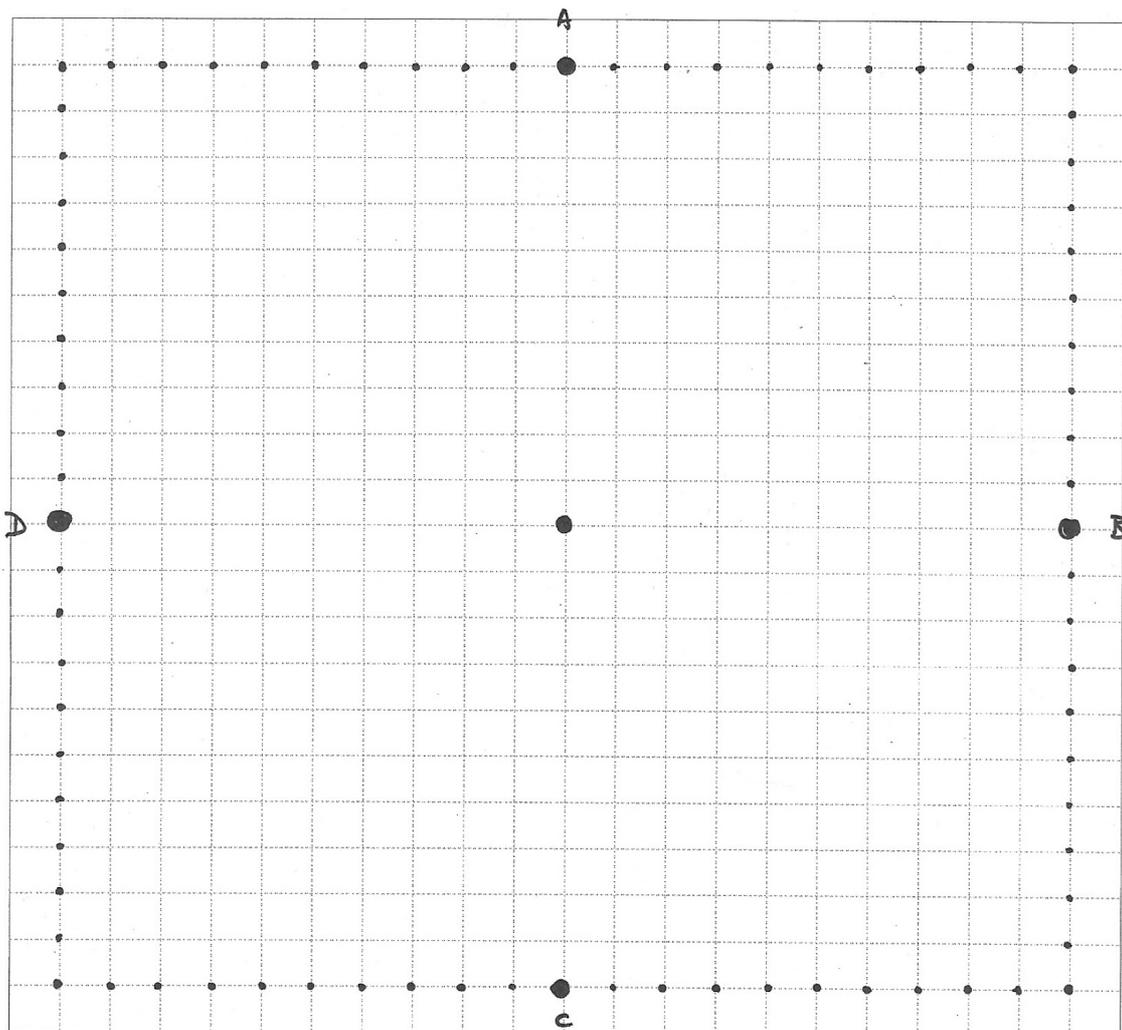
In this task, you will learn how a routine topic in coordinate geometry is, with the aid of graphing technology, extended to explore deeper concepts of variable, function and transformation. It is based on a craft art idea popular in the 1960's and 1970's. (Search Youtube for "String Art").

Step 1: Construct the string art design manually

Question 1.

Complete a manual string art design of a circle as per teacher instruction on the grid below.

- Draw a line from Point A to the second point on the right edge of the square.
- Continue this pattern until the middle of the grid appears to be 'circular'.



Step 2: Construct rules for each line (1st quadrant)

Now suppose we overlay a Cartesian plane, with the origin at the centre, and the grid spacing at 1 unit. For example, Point A has coordinates (0,10) and Point B has coordinates (10,0).

Question 2.

Mark the coordinates of all the relevant points in the first quadrant.

Constructing rules from two points

Recall that for the line with equation $y = mx + c$ passing through the points (x_1, y_1) and (x_2, y_2) , the gradient (m) and the y -intercept (c) can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$ and $c = y_2 - mx_2$

For example, the student work sample shows how the equation of the line joining the points (1,10) and (10, 8) can be found.

$$\begin{aligned} & (1, 10) \quad (10, 8) \\ & y = mx + c \\ & m = \frac{y_2 - y_1}{x_2 - x_1} \quad c = y_2 - mx_2 \\ & = \frac{8 - 10}{10 - 1} \quad = 8 - \left(\frac{-2}{9}\right)(10) \\ & = \frac{-2}{9} \quad = 8 + \frac{20}{9} \\ & \quad \quad = \frac{72}{9} + \frac{20}{9} \\ & \quad \quad = \frac{92}{9} \\ & y = \frac{-2}{9}x + \frac{92}{9} \end{aligned}$$

Question 3.

Using the method shown right, calculate the equation for each line drawn in this quadrant. Write the 10 equations in the space below.

(x_1, y_1)	(x_2, y_2)	Equation
(0,10)	(10,9)	
(1,10)	(10,8)	
(2,10)	(10,7)	
(3,10)	(10,6)	
(4,10)	(10,5)	
(5,10)	(10,4)	
(6,10)	(10,3)	
(7,10)	(10,2)	
(8,10)	(10,1)	
(9,10)	(10,0)	

Step 3: Check your string art design via technology (1st quadrant)

We will now use the TI-Nspire CAS to check the accuracy of calculated lines for the first quadrant of the design.

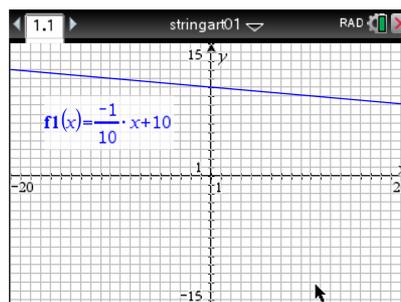
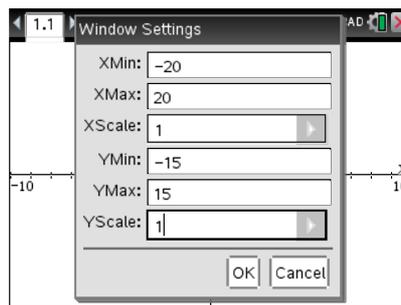
To do this using the TI-Nspire CAS

- Press  > **New Document**, and then select **Add Graphs**.
- Press  > **Window/Zoom** > **Window Settings** to make the graph screen have dimensions $[-20,20]$ by $[-15,15]$, and the **XScale** and **YScale** values as 1 (see correct values shown right).
- Press  > **View** > **Grid** > **Lined Grid** to make a lined grid visible
- Press   and then enter the rule for the first line

Let the first line have equation $y = \frac{-1}{10}x + 10$

The graph screen should appear as at right (Note that the function 'f(x)' is used here instead of 'y')

- To make it easier to view the graphs without the labels cluttering the view, press  > **Settings**, and check the option to 'Automatically hide plot labels'.



Question 4.

Use the above processes to enter and graph the 10 straight lines that you found in Question 3. Make sure that they make a pattern forms similar to the one shown in your manual design from Question 1 (except that the lines continue in both directions here!).

Step 4: Building a set of lines (teacher-led)

Observe the equations of the lines you have constructed – might there be a way to generate the above set of lines by looking at the **set** of values of x_1, x_2, y_1, y_2 being used? Here are the pairs of points for each line found previously.

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9	Line 10
(x_1, y_1)	(0,10)	(1,10)	(2,10)	(3,10)	(4,10)	(5,10)	(6,10)	(7,10)	(8,10)	(9,10)
(x_2, y_2)	(10,9)	(10,8)	(10,7)	(10,6)	(10,5)	(10,4)	(10,3)	(10,2)	(10,1)	(10,0)

Question 5.

What values can x_1, x_2, y_1, y_2 take (write your answers in set notation)?

Question 6.

Press  > **New Document**, and then select **Add Calculator**. Then construct the following variable definitions. Your teacher will lead you through the steps to construct a single general equation which will generate a set of lines.

$$x1:=\{0,1,2,3,4,5,6,7,8,9\}$$

$$y1:=10$$

$$x2:=10$$

$$y2:=\{9,8,7,6,5,4,3,2,1,0\}$$

Step 5: Transforming the lines (teacher led)

Now look at the constructed graphs – how are the parts of the designs that are in the remaining quadrants different to the first quadrant designs?

Question 7.

Observe your manually constructed design from Question 1. Consider the first quadrant line with

equation $y = \frac{-1}{10}x + 10$. What is the ‘mirror image line’ in the second quadrant? What are the

coordinates used to construct the line, and how are they different from those used in the first quadrant line? Try to create 3 new general equations in $f2(x)$, $f3(x)$ and $f4(x)$ that would complete the ‘circle’ design. Explain your reasoning.

$$f2(x) =$$

Reasons:

$$f3(x) =$$

Reasons:

$$f4(x) =$$

Reasons:

Step 6: Extension explorations: How can we change the design?

In this open-ended part of the investigation, explore one or more of the following challenging questions!

Question 8.

Identify the shape that forms the boundary of these 4 sets of straight lines. Is it really a circle? Is it a regular shape?

Question 9.

Modify your design to make a regular octagon.

Question 10.

Make a string art design with a similar shape to the original design, but much smaller?

Question 11.

Make a string art design that is more elliptical (hint: think how you could change the gradients but keeping the same y-intercept)?

Question 12.

Move the original design so it is centred at (2,0)?