Circles on the Complex Plane

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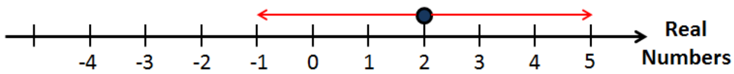
Answers

7 8 9 10 **11** **12**

TI-Nspire Investigation Student 45 min

Introduction

What numbers are 3 units away from the number 2 on the real number line?

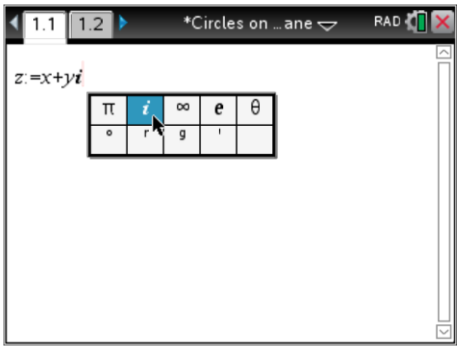


http://bit.ly/ComplexCircles

We can write this as $|x-2|=3$ where x represents the points on the real number line that are 3 units away from the number 2. What happens if the question is changed to include complex numbers? Complex numbers occur on a plane rather than a line. If z is used to represent the set of points that are 3 units away from the number 2, the equation is simply re-written as: $|z-2|=3$. What does this look like?

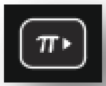
Understanding the Equation

Open the TI-nspire document: "Circles in the Complex Plane". The first page is a calculator application.



Define z as a complex number:

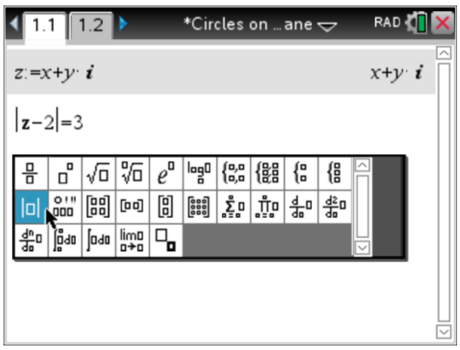
$$z := x + yi$$



The complex number ***i*** can be called up from the maths symbols and constants menu located on the π key. The multiplication sign between the y and ***i*** is implied and will automatically be inserted.

Use the mathematical operations template to locate the absolute value and enter the equation:

$$|z-2|=3$$



The equation will automatically be re-written in Cartesian form. Once the Cartesian equation is displayed, press the x^2 key followed by enter, both sides of the equation will be squared. Compare the answer to the following:

$$(x-2)^2+y^2=9$$

Author: P. Fox

Question: 1.

Describe the set of complex points that are 3 units from the point: z = 2.

Circle with radius 3 centred at z = 2. Equation: $|z-2|=3$ or $(x-2)^2+y^2=9$

Question: 2.

Write an equation for the set of points that would be 3 units from the point z = 2*ı*.

Circle with radius 3 centred at z = 2*ı*. Equation: $|z-2i|=3$ or $x^2+(y-2)^2=9$

Question: 3.

Write an equation for the set of point that would be 5 units from the point: $3+4i$.

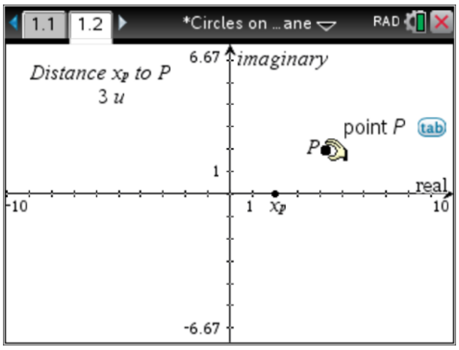
Circle with radius 5 centred at: $z=3+4i$. Equation: $|z-(3+4i)|=5$ or $(x-3)^2+(y-4)^2=25$

Geometric Exploration

Navigate to the Graph application on page 1.2. The point P has been set so that it will remain 3 units from the point x_p.

Move the mouse over point P and grab it by pressing CTRL + Click or by holding the mouse button down for approximately 2 seconds.

Drag point P around the Argand* plane. Notice that the point can only move along a fixed path.



Release point P by pressing **Esc**. Use the **menu** to access the **Trace** option followed by **Geometry Trace**. Click once on point P (this activates the trace feature) then click and grab point P and move it around once again.

Question: 4.

What path is traced out by Point P and what would be the corresponding equation?

Path is a circle (set of points equidistant from z = 2) with equation: $|z-2|=3$

The 'zeros' command finds all points in an expression that equal zero, it can be used to draw the set of points defined by:

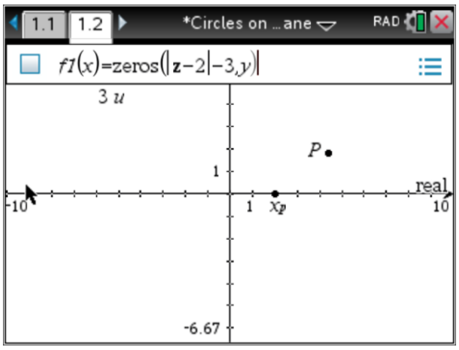
$$|z-2|=3$$

From the above rule it follows that:

$$|z-2|-3=0$$

In the equation entry line type:

$$zeros(|z-2|-3, y)$$



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Question: 5.

Find the points that are 3 units from both: $z = 2 + 2i$ and $z = -2 - 2i$.

This problem can be solved geometrically or algebraically. The algebraic solution is shown here:

$$|z-(2+2i)| = 3 \text{ and } |z-(-2-2i)| = 3$$

$$(x-2)^2 + (y-2)^2 = (x+2)^2 + (y+2)^2$$

$$(y+2)^2 - (y-2)^2 = (x-2)^2 - (x+2)^2$$

$$8y = -8x$$

$$y = -x$$

This represents all points that are equidistant from $z = 2 + 2i$ and $z = -2 - 2i$.

$$|x - xi - (2+2i)| = 3 \quad \textit{Substituting } y = -x$$

$$(x-2)^2 + (-x-2)^2 = 9$$

Therefore: $$2x^2 + 8 = 9$$

$$x = \frac{\pm\sqrt{2}}{2}$$

Complex solutions: $z = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ and $z = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

Question: 6.

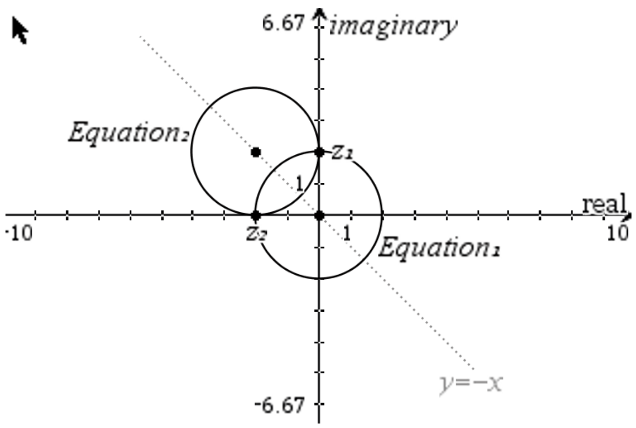
A set of points z_n on the complex plane are located *d* units from the point: $z = a + bi$, two of the points are: $z_1 = 2i$ and $z_2 = -2$.

a. If *d* = 2, determine two possible equations for the set of points.

b. If *d* = 10, determine two possible equations for the set of points.

c. Determine the relationship between a and b for any distance *d*.

This problem can be solved geometrically or algebraically. The geometric solution is shown here:



Equation 1: $|z| = 2$ & Equation 2: $|z-(-2+2i)| = 2$

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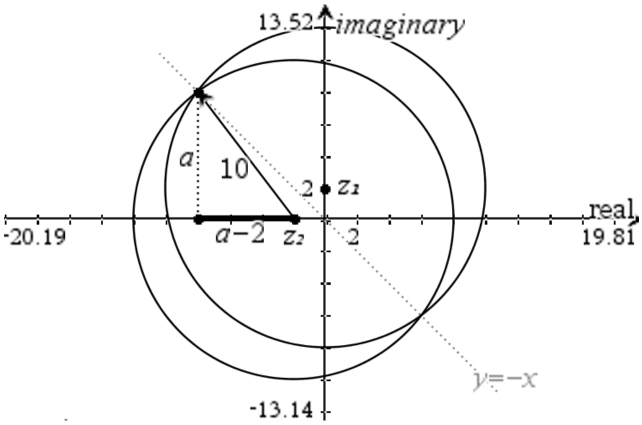
For part (b):



From the diagram the solutions line on the perpendicular bisector of z_1 and z_2:

$$z = -a + ai$$
$$a^2 + (a-2)^2 = 10^2$$
$$a = -6 \quad or \quad a = 8$$
$$z = 6 - 6i \quad or \quad z = -8 + 8i$$

All points (solutions) for *d* would lie on the line: Imag(z)=-real(z) therefore *a* = - *b*

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