Triangle in the Matrix

About the Lesson

This activity is meant to be an introduction for using the matrix formula to find the area of a triangle. Students will verify that the matrix formula works with a right triangle and practice applying the formula with two additional examples. As a result, students will:

- Represent triangles using matrix notation.

- Find the area of a triangle using determinants.

Vocabulary

- matrix formula
- determinant

Teacher Preparation and Notes

- The students should attempt the extension problem algebraically if they know how to calculate the determinant of a 3 × 3 matrix. Another option is to have students create a geometric model of the problem and try various locations along the line $y = 100$.
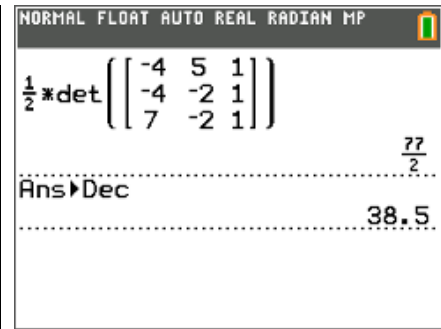
Activity Materials

- Compatible TI Technologies:

    TI-84 Plus*

    TI-84 Plus Silver Edition*

    🌐TI-84 Plus C Silver Edition

    🌐TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.

- Watch for additional Tech Tips throughout the activity for the specific technology you are using.

- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Triangle_in_the_Matrix_Student.pdf
- Triangle_in_the_Matrix_Student.doc

Problem 1 – Testing a Formula

On the worksheet, students are given the formula for determining the area of a triangle using a matrix.

They are told that the straight lines on either side of the matrix represent finding the determinant. Students will need to use the **det** command on the calculator (found by pressing [2nd] [x⁻¹] and select **MATH**).
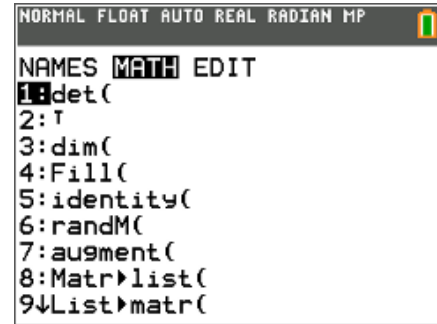
Students can enter the matrix directly on the Home screen. They need to press [alpha] [zoom] and select the number of rows of the matrix; then press [2nd] [▶] to select the number of columns in the matrix. Once the matrix template is entered, the students can enter their values into the matrix by using the arrow keys to move between each element of the matrix. To display the answer as a decimal, press [math], select **dec** and press [enter].
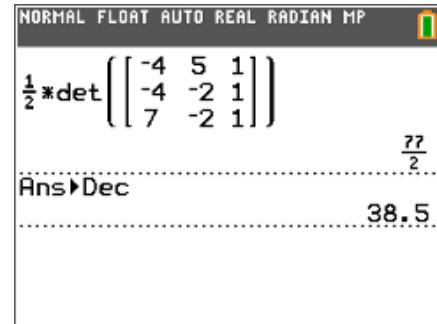
1. What is the area of the triangle using the matrix method?

   Answer: 38.5

2. Check the answer to question 1 by finding the area of the triangle using the geometry formula $A = \frac{1}{2}bh$.

   Answer:
   $$A = \frac{1}{2}bh$$
   $$= \frac{1}{2}(11)(7)$$
   $$= 38.5$$

3. Why do you think that $\pm$ is needed in the matrix method?

   Answer: Depending on the order of the points entered in the matrix, it is possible to have a negative determinant, which would lead to the triangle having a negative area (area can never be negative.) So the $\pm$ is used to ensure the result is always positive.

Have students explore entering the coordinates in the matrix in a different order. They will see it changes the calculation of the determinant from positive to negative, making the $\pm$ symbol necessary.

Problem 2 – Practice Problems

Students will find the area of two non-right triangles using matrices.

Challenge students to find two ways to enter the coordinates into the matrix so that the determinant gives both a positive and a negative result.
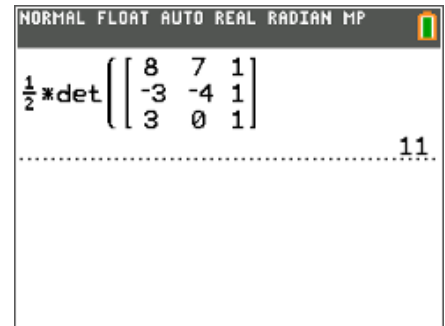

Tech Tip: Students can be more efficient in their calculator usage, by using the up arrow to select a previous calculation, then pressing enter to copy the selection to the entry line. Now the students can edit the copied entry by using the arrow keys.

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4. Find the area of the triangle to the right using matrices.

Answer: $\dfrac{1}{2}\det\begin{bmatrix} 8 & 7 & 1 \\ -3 & -4 & 1 \\ 3 & 0 & 1 \end{bmatrix} = 11$

5. Check the calculated area by using geometric formulas or by estimating the number of squares inside of the triangle.
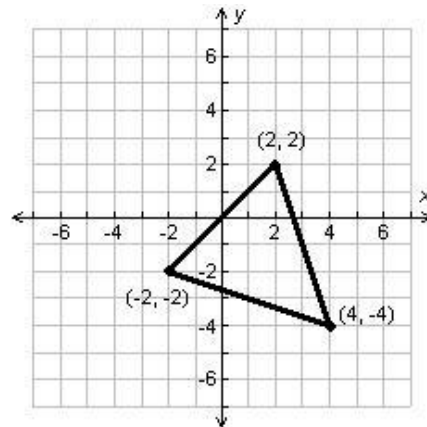
Answer: 11
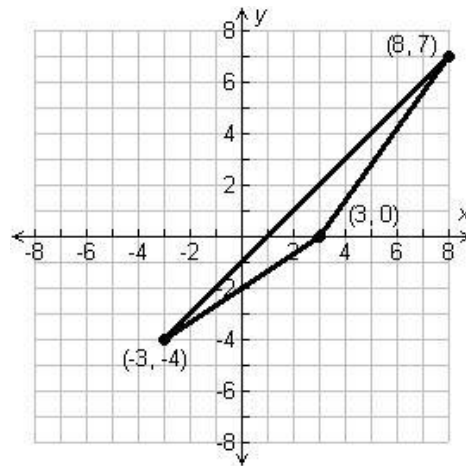
6. Find the area of the triangle to the right using matrices.

Answer: $\dfrac{1}{2}\det\begin{bmatrix} 2 & 2 & 1 \\ -2 & -2 & 1 \\ 4 & -4 & 1 \end{bmatrix} = 16$

7. Check the calculated area by using geometric formulas or by estimating the number of squares inside of the triangle.
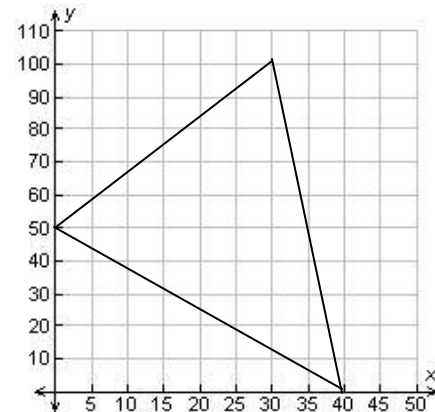
Answer: 16

Extension – Fencing a Garden

Students are to find the point on $y = 100$ where the gardener should place the post.

This will be done through trial and error, although they should know that the y-coordinate does not change.

The correct answer is to build it at (30, 100).

8. A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and (40, 0). The final fence post is on the property line at $y = 100$. Find the point where the gardener can place the final fence post.

Answer:

$$1750 = \pm\frac{1}{2}\begin{vmatrix} 0 & 50 & 1 \\ 40 & 0 & 1 \\ x & 100 & 1 \end{vmatrix}$$

$$1750 = \pm\frac{1}{2}\left(0\begin{vmatrix} 0 & 1 \\ 100 & 1 \end{vmatrix} - 50\begin{vmatrix} 40 & 1 \\ x & 1 \end{vmatrix} + 1\begin{vmatrix} 40 & 0 \\ x & 100 \end{vmatrix} \right)$$

$$1750 = \pm\frac{1}{2}(-50 \cdot x - 50 \cdot 40 + 4000)$$

$$1750 = \pm(-25x - 1000)$$

$$25x = 750$$

$$x = 30$$