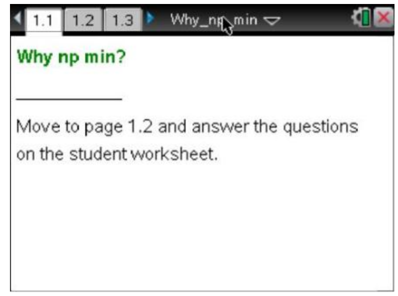
Why np min?

Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Why_np_min.tns.*



This lesson examines similarities of, and differences between, binomial and normal distributions, with a focus on approximating binomial distributions with normal distributions.

1. You have already studied binomial random variables. This question reviews important facts about the distributions of such random variables.
   a. Describe exactly what a binomial random variable measures (counts).

   b. Write the formula for the binomial probability of exactly x successes. Be sure to state clearly what each letter in your formula represents.

   c. In the context of a binomial scenario, what do the values of np and $n(1 - p)$ mean?

Move to page 1.2.

2. The image on Page 1.2 is the probability distribution function (*pdf*) for a binomially-distributed random variable defined by n trials and probability p of success on each trial.
   a. Explain why the graph consists of a set of separated vertical bars.

   b. What does the height of each bar represent, and what formula is used to calculate that height?

3. A description of any distribution usually includes information about its center and shape, among other things. The questions below ask you to use sliders to investigate how values of ***n*** and ***p*** affect the center and shape of binomial distributions.
   a. Set ***n*** to 30. Then vary ***p*** from 0 to 1. How are ***np*** and ***n***(1 – ***p***) related to the “center” of the distribution?

   b. Shape usually includes both modality (unimodal, bimodal, multimodal) and symmetry (symmetric, skewed left, skewed right). Summarize your observations regarding shape as you changed values of ***p*** in question 3a.

   c. Repeat questions 3a and b with other values for ***n***. (You might need to adjust ***p*** more slowly as its value nears 0 or 1 in order to observe the changes more clearly.)

   d. Explain why some choices of ***n*** and ***p*** lead to graphs that are very non-symmetric.

Move to page 1.3.

4. The plot on Page 1.3 contains exactly the same information as that on Page 1.2. However, the appearance of the plot is different and probably looks more familiar. Describe the differences between these two plots, and then comment on their relative advantages.

5. Select a value of your own choosing for ***n***. Then choose ***p*** as far from 0.5 as you can while still displaying a probability histogram that appears reasonably symmetric. Record your values for ***n*** and ***p***.

6. You found values for ***n*** and ***p*** in question 5 that produce an approximately symmetric binomial probability histogram. Thus it might seem reasonable to try to overlay a normal curve over your graph. But which one?
    a. What two numbers (parameters) are needed in order to graph any specific normal curve?

    b. What are the mean and standard deviation of the binomial *pdf* you made in question 5? Show the formulas you use to calculate these values.

    c. Explain why knowing just ***n*** and ***p*** is sufficient to define both the binomial *pdf* and its related normal *pdf*.

Move to page 2.1.

7. The plot on Page 2.1 graphs a normal *pdf* on top of the corresponding binomial probability histogram. Set the controls of Page 2.1 to match your graph in question 5.
    a. Describe how well the normal *pdf* fits the binomial probability histogram.

    b. Look back at your answer to Question 3d. Based on what you saw earlier, predict exactly how the graph of a normal *pdf* might differ from its corresponding binomial *pdf* if the average number of successes or failures is very small. Be as specific as you can in describing the graphs. Remember, normal *pdf*s are *always* symmetric.

    c. Use the sliders to check your predictions. Describe your observations.

Move to page 3.1.

8. a. What are the possible values for a binomial random variable (count of successes)?

   b. Explain why your answer to part a is problematic when using a normal distribution to approximate a binomial distribution.

   c. Page 3.1 displays the normal probabilities for $x < 0$ and $x > n$. For the actual binomial distribution, what are the exact values of these two probabilities? In general, how do the normal and binomial probabilities compare?

   d. Observe these two probabilities as you vary $\boldsymbol{n}$ and $\boldsymbol{p}$. Record the combinations of $\boldsymbol{n}$ and $\boldsymbol{p}$ for which the normal approximation gives a value of at least 0.01 for either of these two “extreme” probabilities. Be sure to check for values of $\boldsymbol{p}$ near 1 as well as for values near 0. (Note: Use the unlabeled clicker to the left of the vertical axis to hide/show the binomial histogram bars.)

9. Many textbooks include guidelines for using normal random variables to approximate binomial random variables. Those guidelines usually are of the form, “Check that each of the values of np and $n(1 – p)$ is at least ____ .” Different books disagree on what they prefer to put in the blank as their “magic number.”

   a. What number would you put there, and why?

   b. How does the quality of the approximation of a binomial distribution by a normal distribution vary as the “magic number” increases? Explain.