Taylor Made Polynomials

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Student Activity

7 8 9 10 **11** **12**

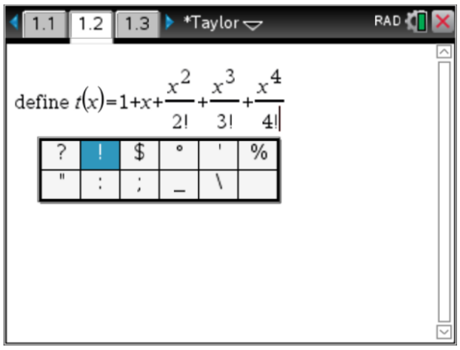
TI-Nspire | Investigation | Student | 45 min

Introduction

Brook Taylor (1685 – 1731) was an English Mathematician, he is acknowledged as formerly introducing the Taylor Series which built on the work of Scottish Mathematician James Gregory (1638 – 1675). The Taylor Series is the respresentation of a function as an infinite sum of terms. A Taylor polynomial is a finite number of terms that can be used to approximate the function. In this activity we will use Taylor Polynomials, but understand that the same ideas can be applied to the series.

Instructions

Open the TI-Nspire file: **Taylor Made**



Navigate to page 1.2 and define the following polynomial:

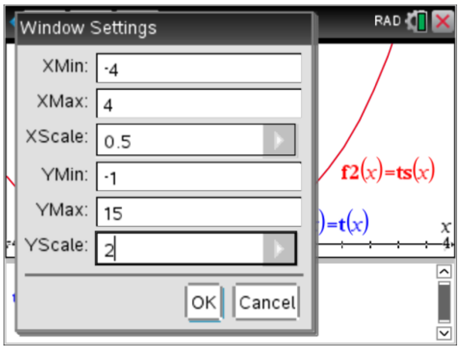
$$t(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

Navigate to page 1.3 to see a graph of the polynomial: $t(x)$

Question: 1.

Describe the shape of the graph looking through the Window settings provided on Page 1.3.

The polynomial function (below) has been defined:



$$ts(x) = \sum_{n=0}^{m} \frac{x^n}{n!}$$

Change the window settings in the **Graph** application to match the settings shown then graph this function: $ts(x)$.

This new function contains a parameter (m). The slider value has been set to match: $t(x)$. Increase the value of m using the slider.

Question: 2.

Explain how the polynomial definition for $ts(x)$ works. (Use example values of m)

Question: 3.

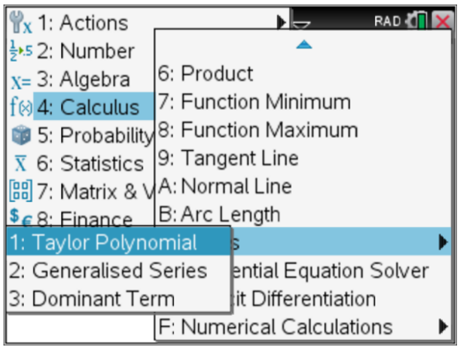
Successively increase the value for m (slider) and observe the changes to the graph. What function is this Taylor Polynomial modelling?

Question: 4.

Determine the derivative of this polynomial and explain how this supports your previous answer.

Author: P. Fox

Modulus and Argument



Navigate to page 2.1.

To generate a Taylor Polynomial use the calculus menu:

Calculus > **Series** > **Taylor Polynomial**

The syntax for the command is as follows:

Taylor(Expression, Variable, Order)

Question: 5.

Generate the following Taylor Polynomials:

i. Taylor(e^x , x, 8)

ii. Taylor(cos(x), x, 8)

iii. Taylor(sin(x), x, 8)

iv. Taylor(cos(x) + sin(x), x, 8)

Question: 6.

Compare answers (i) and (iv) above.

Question: 7.

Determine each of the following:

i. Taylor(e^{ix} , x, 8) [Make sure to use the complex number i.]

ii. Real terms in: Taylor(e^{ix} , x, 8)

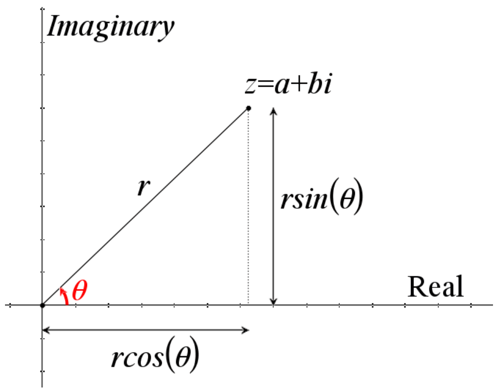
iii. Imaginary terms in: Taylor(e^{ix} , x, 8)

Question: 8.

For each of the following $z=\sqrt{3}+i$

i. Calculate z^2

ii. Calculate z^3



Question: 9.

Use the diagram (right) to show that the complex number:

$z=a+bi$ (rectangular form) can be written in polar form:

$z=r\left(\cos(\theta)+i\sin(\theta)\right)$

Question: 10.

Explain the connection between: $z=re^{i\theta}$ and $z=rcis(\theta)$

Question: 11.

Use the index laws to show that $z^n=r^n cis(n\theta)$

Question: 12.

Write $z=\sqrt{3}+i$ in polar form (either $z=re^{i\theta}$ or $z=rcis(\theta)$) and hence evaluate z^2, z^3 and z^6.

Author: P. Fox