Investigation of End Behavior

ID: 10250

Time required
45 minutes

Activity Overview

Students explore end behavior of rational functions graphically, algebraically, and by using tables. They will use multiple representations to look at values a given function approaches as the independent variable goes to positive or negative infinity. Tools are provided which support them in using a graphical approach, evaluating functions at different values, and using a function equation to find the end behavior of a rational function.

Topic: Rational Functions & Equations

- *Graph a rational function to verify its domain and range.*
- *Evaluate a rational function of x at any value of x.*

Teacher Preparation and Notes

- *This investigation is intended to be used in an advanced Algebra 1 class as an introduction to the concept of end behavior with rational functions. It could also be used in an Algebra 2 class as a review of end behavior or with students who might be struggling with some of these concepts.*
- *This activity is initially **teacher-led**, followed by students working individually or with a partner. You may use the following pages to present the material to the class and encourage discussion; students should follow along using their calculators.*
- *It would be beneficial (but not necessary) for students to have had an introduction to rational functions before beginning this activity. This activity is a great lead-in to finding end behavior by polynomial long division.*
- *Information for an optional extension is provided at the end of this activity; information for students is provided on the student worksheet. Should you not wish students to complete the extension, you may have students disregard that portion of the student worksheet.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10250" in the keyword search box.***

Associated Materials

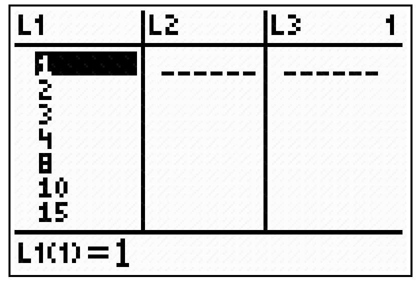
- *EndBehavior_Student.doc*

Problem 1 – Cost per person for a pizza order

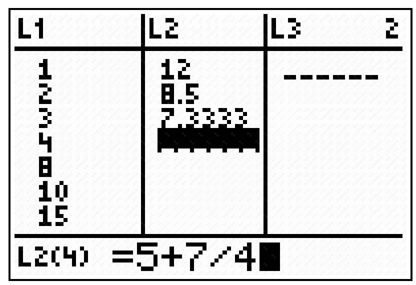
The coach of the football team wants to order individual pizzas to eat after their game. Pizza-To-Go charges $5 for each individual pizza, plus an overall delivery charge of $7. The coach needs to figure out the cost per player so that each player who wants pizza can contribute enough to cover the total cost.

Players	1	2	3	4	8	10	15	25

Instruct students to enter the data shown into their calculator. Press [STAT] [ENTER] to open the **List Editor** screen. Enter the number of players into **L1**.

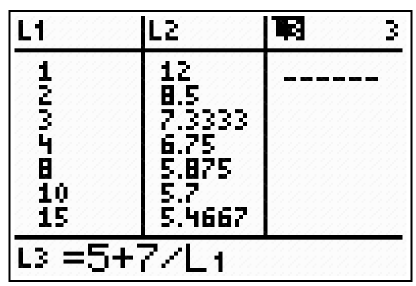


Students will calculate the average cost per player in **L2**, as shown. Have them use these values given to generate values in this column using what they know about the problem. Students should enter an expression for each entry, such as **5+7/1**, **5+7/2**, **5+7/3**, etc. They should find that the cost does indeed decrease as more students order pizza.



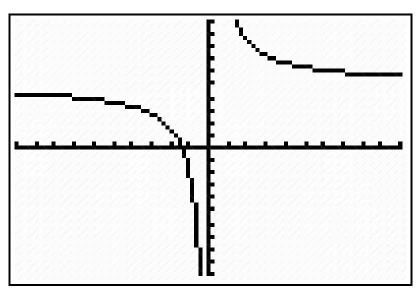
Students should now write a general equation based on the recursive process they just used. They can then test their equation in **L3**. They can press [2nd] [L2] to type **L2**.

An appropriate equation is: $\mathbf{L3} = 5 + \frac{7}{\mathbf{L1}}$



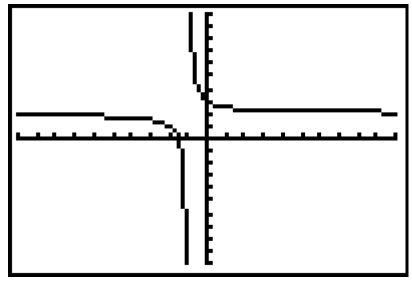
If their calculations are correct, the entries in **L2** and **L3** will be identical.

They graph the function in **Y1**. What value does the graph approach, as *x* gets larger and larger? They should adjust the window settings if necessary to see, and then answer the questions on their worksheet.



Problem 2 – Investigating end behavior

In **Y1**, students graph $f(x) = \dfrac{2x+3}{x+1}$.



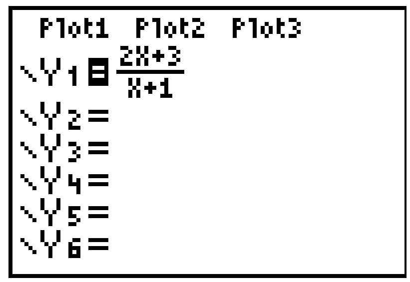
Have students change the window so that the *x*-axis goes from –500 to 500. What happens to the graph?

They should see that the graph of the function ends up "looking like" a line. So as the *x*-values get very large in magnitude, the graph "looks" like its end behavior, which here is the line $y = 2$.

If using Mathprint OS:

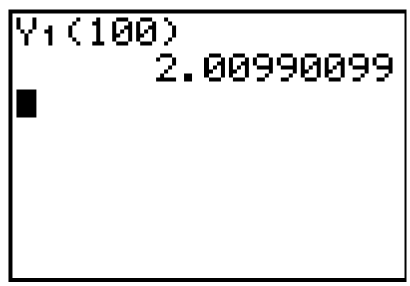
Students can display fractions in the Y= screen. To do this, press [ALPHA] [F1] and select **n/d**. Then enter the value of the numerator, press [▼] to move to the bottom of the fraction, and enter the value of the denominator. Press [ENTER].

Note: Parentheses are not needed in the numerator or the denominator.



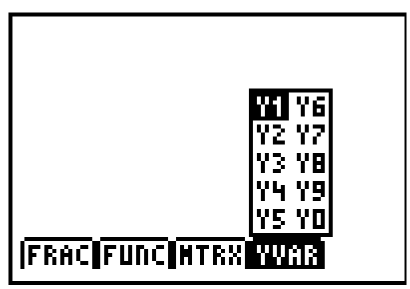
Next, students will investigate this behavior using their calculator and a table. Prompt them use 10, 100, and 1000 and then –10, –100, and –1000 to simulate the *x*-values going to positive and negative infinity, respectively. To calculate the value of **Y1**(*x*) for an *x*-value, Press [VARS], go to **Y-VARS** and select **1:Function…**.

Choose **Y1** from the list. Type the value of X, then press [ENTER].

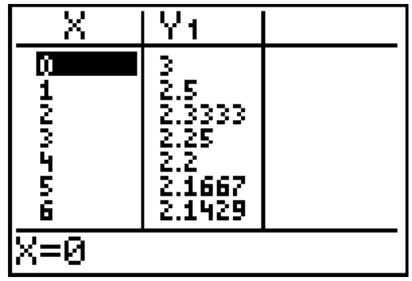


If using Mathprint OS:

Students use the shortcut menu to enter Y1 from the Home screen. To do this press [ALPHA] [F4] and select **Y1**.



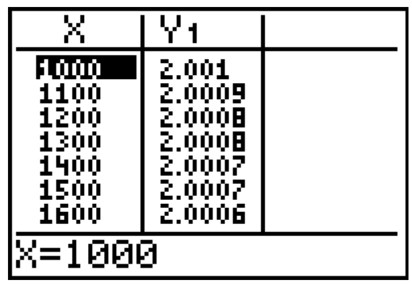
To view a table of values, press STAT ENTER and arrow up and down the list.



To change the *x*-values, press 2nd [TBLSET] and adjust the settings there.

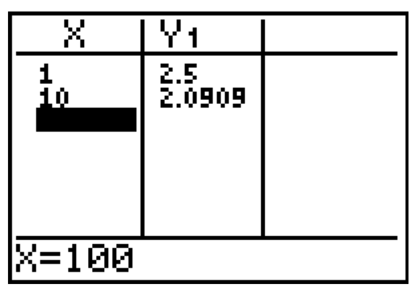
TblStart determines the first *x*-value in the table and **ΔTbl** determines how much the *x*-values in the table increase from one row to the next.

For example, this table has a **TblStart** of 1000 and a **ΔTbl** of 100.



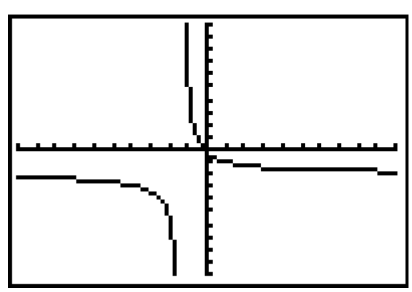
Students can also set the table settings to ask for the independent (*x*) value.

Then they can enter any *x*-value they wish. Some values to try are –1, –10, –100, and –1000 to simulate approaching negative infinity and then 1, 10, 100, and 1000 to simulate approaching positive infinity.

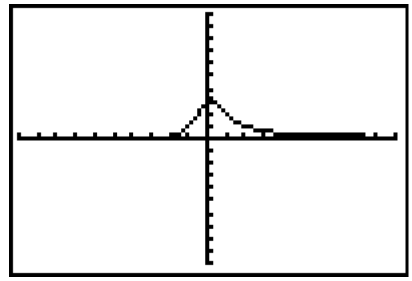


Students graph another rational function $g(x) = \frac{-6x - 1}{3x + 4}$ in **Y1**.

They change the viewing window as before. Ask students what they observe. They investigate the behavior of *g*(*x*) using either by calculating values of **Y1** or viewing a table of values.



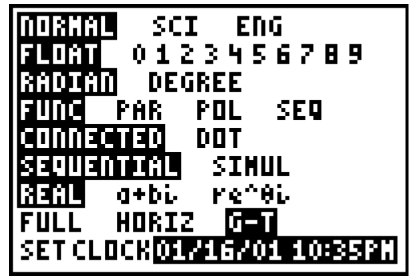
Graph $h(x)=\dfrac{x+3}{x^2+1}$ in **Y1**.



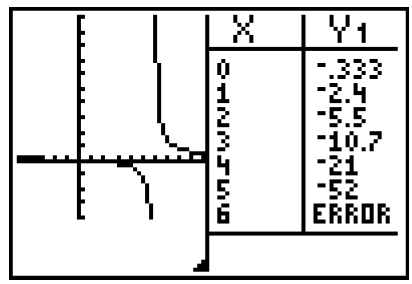
Have students adjust the window as done in previous exercises. Ask them what they observe.

Students investigate the behavior of *h*(*x*) as before. Here, they should determine that the function approaches $y = 0$.

To view a table of values and the graph simultaneously, press MODE and press ENTER when **G-T** (Graph Table) in the second to last row is highlighted.

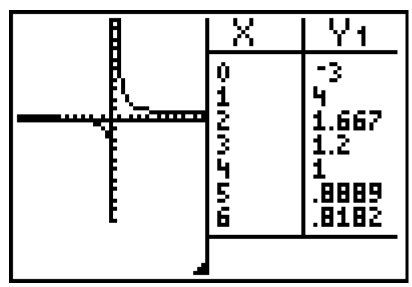


Graph $j(x)=\dfrac{10x+2}{x-6}$ in **Y1**. (They will need to adjust the window to view the shape of the graph.)



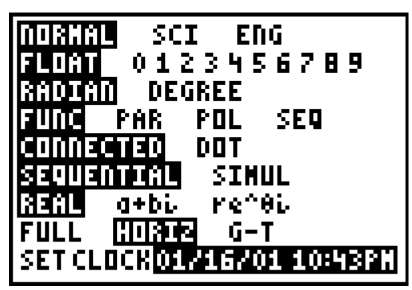
They view a table of values to investigate the values of *j*(*x*) as *x* gets larger and smaller. Students will see that the end behavior of the function is $y = 10$.

Graph $k(x)=\dfrac{x+3}{2x-1}$ in **Y1**.

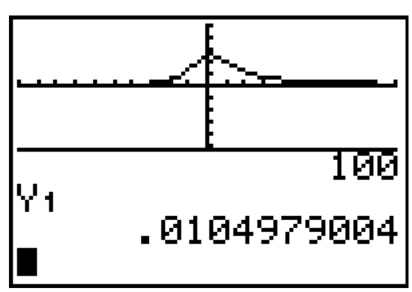


Students will similarly explore then end behavior of the function. Here, they should determine that the function approaches $y=\dfrac{1}{2}$.

To view Home screen calculations and a graph simultaneously, press MODE and press ENTER when **Horiz** (Horizontal Split) in the second to last row is highlighted.



Graph $m(x)=\dfrac{x+5}{x^2+2}$ in **Y1**.



Calculate values of **Y1** on the Home screen to investigate the values of *m*(*x*) as *x* gets larger and smaller.

Instruct students to complete the remainder of their worksheet.

Extension

Students should graph and examine the end behavior of the function $f(x) = \frac{(x)(x+3)}{x+2}$. This function has an end behavior asymptote of *y* = *x*. This will introduce them to the idea that not all rational functions have horizontal end asymptotes. You can suggest they try different functions and asymptotes by altering the definition of **Y1**.

