Math Objectives

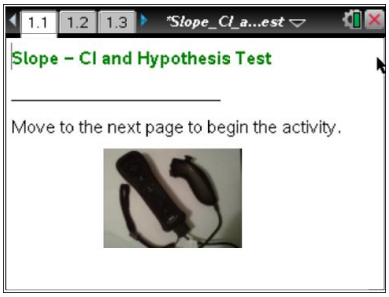
- Students will be able to identify the conditions that need to be met to perform inference procedures for the slope of a regression line.
- Students will be able to interpret the results of a hypothesis test for slope and write the conclusion in context.
- Students will be able to calculate the degrees of freedom for inference procedures on bivariate data.
- Students will be able to interpret a confidence interval for the slope in context.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

Vocabulary

- bivariate data
- confidence interval
- degrees of freedom
- hypothesis test for slope
- least-squares regression
- normal probability plot
- *p*-value
- residual
- scatterplot
- slope

About the Lesson

- This lesson involves investigating the confidence interval and hypothesis test for the slope of a regression line.
- As a result, students will:
  - Check the conditions for inference for slope by inspecting a scatterplot of the data, the normal probability plot, and a plot of the residuals.
  - Write the appropriate null and alternative hypotheses for the given scenario, determine the degrees of freedom for the data, and interpret the results of a hypothesis test for the slope.
  - Analyze the results of a hypothesis test performed on the TI-Nspire, make a decision to reject or fail to reject the null hypothesis, and write their conclusion in the context of the situation.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Slope_CI_and_Test_Student.pdf
Slope_CI_and_Test_Student.doc
Slope_CI_and_Test_Create.doc

TI-Nspire document

Slope_CI_and_Test.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

- Calculate and interpret a confidence interval in context.
- Compare the information they get about the relationship from a hypothesis test and from a confidence interval.

TI-Nspire™ Navigator™ System

- Use Screen Capture to monitor student progress in creating the .tns file.
- Use Quick Poll to check for student understanding.
- Use Live Presenter to demonstrate how to check the conditions of a hypothesis test.

Prerequisite Knowledge

Students should be familiar with hypothesis tests:

- writing null and alternative hypotheses
- finding a test statistic and *p*-value
- making a decision based on the results

In addition, they should be familiar with creating a scatterplot a residual plot, and a normal probability plot on the TI-Nspire.

- If not, please reference the TI-Nspire Create documents *Scatterplots Pulse Rates* and *Normal Probability Plots*.

Discussion Points and Possible Answers

Teacher Tip: Students can create the .tns file following the steps in the *Slope_CI_and_Test_Create.doc* document, or they can use the premade file, *Slope_CI_and_Test.tns*.

A school district wants to see if there is an association between a student's GPA and the number of hours spent playing video games per week. The data from 14 randomly selected high school students are displayed below.

Student	Hours of Video Game Play per Week	GPA
A	17.5	2.0
B	15	3.0
C	25	0.8
D	15.5	2.7
E	11	2.6
F	12.5	3.6
G	3.5	4.0
H	6	3.2
I	24	1.0
J	13	2.9
K	20.5	1.8
L	9	3.5
M	23.5	1.5
N	2	3.8

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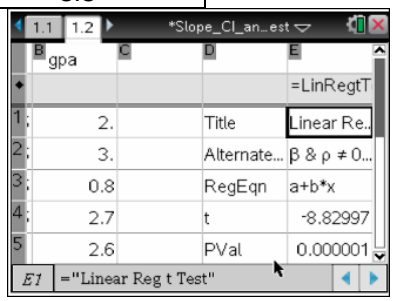

1. Why might the district be interested in determining if there is an association between video game playing and GPA?

   Sample Answers: If students are spending a lot of hours playing video games, they are not studying during those hours. Maybe evidence of an association would provide an incentive for parents, families, and/or teachers to encourage less video game playing and more studying.

2. a. What conditions must be checked in order to perform an inference procedure?

   Sample Answers: A scatterplot of the data should be approximately linear; there should be no pattern in the residuals or the variability of the residuals; the residuals should be approximately normal.

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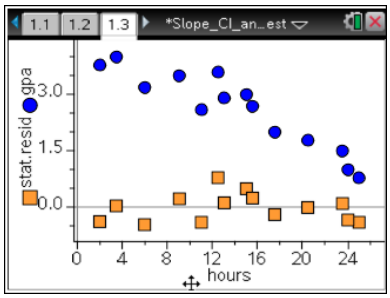

b. Page 1.3 shows the scatterplot of GPA vs. number of hours of video games played per week and the residual plot for the least-squares regression line. Do these plots meet the conditions for a hypothesis test on the slope? Explain your reasoning.

Sample Answers: Yes, the scatterplot shows the data are approximately linear and negatively associated; the residual plot has no pattern and no visible pattern in the variability.

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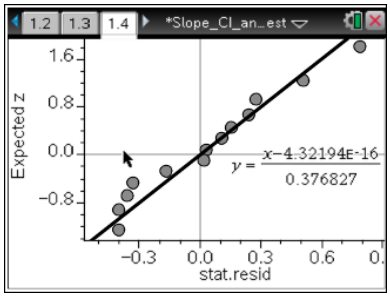

c. Page 1.4 displays the normal probability plot of the residuals. Are the conditions satisfied? Explain your reasoning.

Sample Answers: The normal probability plot of the residuals is approximately linear with no pattern in variability indicating approximate normality. Thus, all conditions seem to be met for the hypothesis test.

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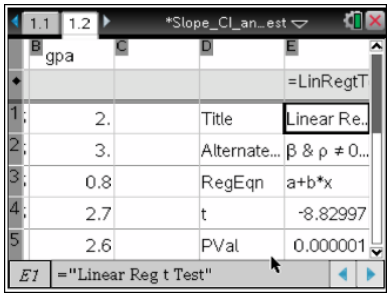

3. a. State the null and alternative hypotheses for the slope (both numerically and in words) to determine whether there is a linear association between the variables.

Sample Answers: H_o: $\beta = 0$; there is no useful linear association between the number of hours of video games played per week and GPA.
H_a: $\beta \neq 0$; there is a useful linear association between the number of hours of video games played and GPA.

b. The results of the hypothesis test on slope are displayed in the spreadsheet. The degrees of freedom for a hypothesis test for slope is found by subtracting two from the sample size (n-2). How many degrees of freedom should be used in this test?

Answer: 14 - 2 = 12 degrees of freedom

c. What is the *t*-test statistic and *p*-value for the hypothesis test? Explain the meaning of the *p*-value.

Answer: The test statistic is -8.83, and the corresponding *p*-value is 0.000001. The *p*-value is the probability of obtaining a test statistic as extreme or more extreme than -8.83 if there was no linear association between number of hours of video games and GPA.

d. Make a decision to reject or fail to reject your null hypothesis using an alpha value of 0.01. Write your conclusion in context.

Sample Answers: I would reject the null hypothesis because the *p*-value is less than the alpha value. Based on this random sample of students, there is strong evidence of a linear association between the number of hours of video games played per week and a student's GPA at the 1% level.

e. If the hypothesis test were changed to determine whether the variables have a useful negative linear relationship, how would the alternative hypothesis and the *p*-value change? Explain your reasoning.

Sample Answers: The alternative hypothesis would state that $\beta<0$. Since we are trying to determine if there is a negative linear association, then the slope should be significantly negative. The *p*-value would be halved (.000001÷2). Since a two-tailed test doubles the *p*-value, because the rejection region would be divided evenly between the two tails, a one-tailed test would be half of a two-tailed *p*-value.

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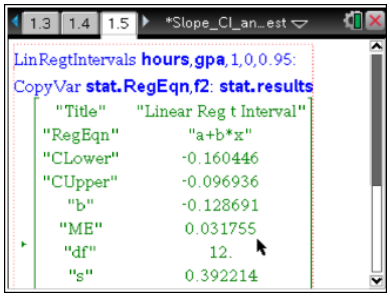

4. The page shows the calculations of a 95% confidence interval for the slope.
   a. Show two ways the interval can be calculated based on the output.

Sample Answers: The confidence interval for slope is found by finding the (sample slope)±(margin of error) or -0.128691±0.031755. Another way is to calculate the interval is (sample slope)±t(standard error) or -0.128691± t (0.014574). The *t*-value is not given in the output.

   b. What is the 95% confidence interval for the slope? Explain what it means in context.

Sample Answers: (-0.16, -0.10). We are 95% confident that GPA decreases, on average, between 0.16 and 0.10 points for each hour of playing video games per week.

   c. Does this confirm that playing video games causes your GPA to go down? Explain your reasoning.

Sample Answers: No, even though we have found that the two variables are negatively associated, it does not mean that the change in one variable causes a change in the other because a causal relationship can only be shown through an experiment.

5. a. How do the results of the hypothesis test compare to the confidence interval?

Sample Answers: The hypothesis test gives evidence that a useful linear relationship exists between the two variables while the confidence interval gives information about the direction of the linear relationship (positive or negative) pertaining to the slope.

   b. Your job is to report the results of your research back to the school board. You only have time to explain either the results of the hypothesis test or the confidence interval. Which do you think provides the most valuable information? Explain your reasoning.

Sample Answers: Some students might argue that the hypothesis test is more useful because it shows that there is evidence that a useful linear relationship exists, while others prefer the confidence interval because it gives information about the rate of change in GPA to the hours of video game playing.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The results of a hypothesis test and confidence interval for slope in context.
- How to calculate the degrees of freedom for a hypothesis test on the slope for linear regression.
- How to make a decision to reject or fail to reject the null hypothesis based on the *p*-value and how to write the conclusion in context.

Assessment

Decide whether the following statements are always, sometimes or never true.
Explain your reasoning in each case.

1. The null hypothesis in a hypothesis test for slope states that there is a linear relationship between two variables.

   Sample Answers: Never true—the null hypothesis always states that there is no relationship between the two variables. It assumes the status quo rather than what is being tested.

2. A *p*-value from a one-tailed test is doubled for a two-tailed test based on the same data.

   Sample Answers: Always true—a *p*-value is the probability of getting a test statistic as extreme as the one obtained or more extreme. If the null hypothesis is true for a two-tailed test, you consider the opposite extreme also, so the p-value is doubled.

3. The degrees of freedom is $n - 1$, where n is the number of ordered pairs, for inference procedures involving bivariate data.

   Sample Answers: Never true—the number of degrees of freedom for inference procedures involving bivariate data is $n - 2$.

4. A 99% confidence interval for a slope that yielded (-0.25, 5.81) shows no significant linear relationship between the two variables.

   Sample Answers: Sometimes true – at a 1% alpha level this is true, but at 5% or 10% it might not be true. A larger confidence level yields a larger interval, thus 99% could contain 0 while a lower confidence interval might not, for example (0.2, 5.36).

TI-Nspire Navigator

Name of Feature: Live Presenter
Live Presenter might be used to have a student demonstrate how to build the .tns file