Secants, Tangents, and Arcs

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will understand that if the intersection point *P* of two lines lies inside a circle, then the measure of the angle formed by the two secants is equal to the average of the measures of the arcs intercepted by that angle and its corresponding vertical angle.
- Students will understand that if the intersection point *P* of two lines lies outside a circle, then the measure of the angle formed is equal to half of the difference of the measures of the arcs intercepted by that angle and its corresponding vertical angle, whether the lines are both secants, a secant and a tangent, or two tangents.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- secant line and chord
- tangent line
- central angle
- intercepted arc

About the Lesson

- This lesson involves discovering the relationship between the measure of the angle of intersection of two lines and the measures of the intercepted arcs for several cases in which two intersecting lines intersect a circle.
- As a result students will:
  - Observe that the angle of intersection has a measure equal to one-half the sum of the measures of the intercepted arcs whenever two secants intersect anywhere in the interior of a circle.
  - Observe that the angle of intersection has a measure equal to one-half the difference of the measures of the intercepted arcs whenever two secants intersect outside of a circle, a secant and a tangent intersect, or two tangent lines intersect.

TI-Nspire™ Navigator™ System

- Quick Poll
- Screen Capture

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Secants_Tangents_and_Arcs_Student.pdf

Secants_Tangents_and_Arcs_Student.doc

TI-Nspire document

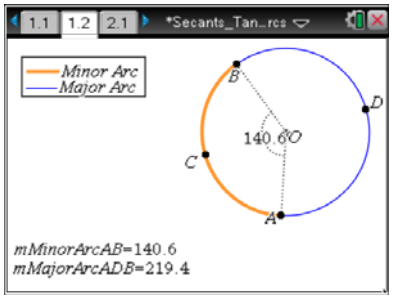
Secants_Tangents_and_Arcs.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜). Press ctrl ☜ to grab the point and close the hand (✊).After the point has been moved, press esc to release the point.

Move to page 1.2.



1. Two points, *A* and *B,* on a circle determine a minor arc *AB*. The measures of ∠*AOB* and the minor arc *AB* are shown. Drag point *A* or *B.* The measure of a minor arc is defined to be equal to the measure of its central angle.

   a. How do you find the measure of the major arc *ADB* if the measure of the minor arc *AB* is known?

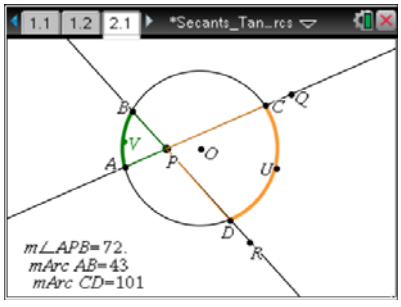
   Answer: If you subtract the measure of the minor arc from 360º, the result is the measure of the major arc.

   b. When are the measures of arcs *AB* and *ADB* exactly the same?

   Answer: Only when *A* and *B* are the endpoints of a diameter of the circle. In this case, both arcs measure 180º and are called semicircles.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Move to page 2.1.



2. The measures of ∠*APB* and the arcs *AB* and *CD* intercepted by ∠*APB* and its vertical angle are shown.

   a. Drag point *P* around in the interior of the circle, but not at the center. With respect to the circle, what types of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

   Answer: Secant lines

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

b. Find a relationship between the measure of $\angle APB$ and the measures of intercepted arcs *AB* and *CD*.

Answer: Half of the sum of the measures of the arcs equals the measure of $\angle APB$.

TI-Nspire Navigator Opportunity: *Screen Capture* and *Quick Poll*
See Note 3 at the end of this lesson.

Teacher Tip: If students are struggling with finding a relationship between the measure of the angle between the secant lines and the intercepted angles, you might remind them that many of the special relationships between measurements in geometry are expressed in terms of arithmetic: angle SUM of a triangle, RATIO of side lengths in similar triangles, and so on. So thinking about simple sums, differences, products, and ratios would be sensible.

c. Drag point *P* to coincide with the center of the circle. With respect to the circle, what types of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

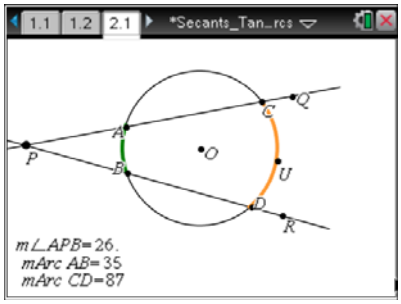
Answer: Secant lines that contain diameters

d. Does the relationship you found in question 2b still hold?

Answer: The relationship still holds, but in this case the arcs are also equal to the measure of $\angle APB$ since it is now a central angle.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.

3. Drag point *P* to several places outside of the circle, but make sure both $\overleftrightarrow{PQ}$ and $\overleftrightarrow{PR}$ still intersect the circle in two points.
   a. With respect to the circle, what types of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?



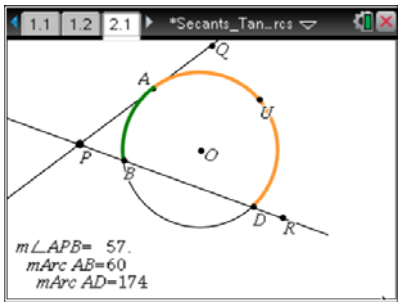
Answer: Secant lines

b. Find a relationship between the measure of $\angle APB$ and the measures of intercepted arcs *AB* and *CD.*

Answer: Half of the difference of the measures of the arcs equals the measure of $\angle APB$.

TI-Nspire Navigator Opportunity: *Screen Capture* and *Quick Poll*
See Note 5 at the end of this lesson.

4. Leave *P* outside of the circle. Make sure $\overleftrightarrow{PQ}$ and $\overleftrightarrow{PR}$ both intersect the circle in two points. Drag point *Q* so that point *C* coincides with point *A*.



a. With respect to the circle, what types of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

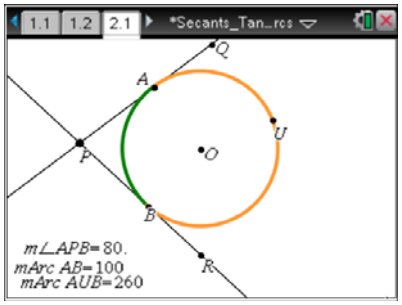
Answer: $\overleftrightarrow{PA}$ is a tangent line and $\overleftrightarrow{PB}$ is a secant line.

b. Does the relationship you found in question 3b still hold?

Answer: Yes, the relationship still holds.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 6 at the end of this lesson.

5. Leave *P* outside of the circle and leave *C* so that it still coincides with point *A*. Drag point *R* so that point *B* coincides with point *D*.



a. With respect to the circle, what types of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

Answer: They are tangent lines.

b. Is there a relationship between the measure of $\angle APB$ and the measure of the minor arc? Does the relationship you found in question 3b still hold?

Answer: The measure of the angle and the minor arc are supplementary (sum to 180˚). Yes, the relationship still holds.

Teacher Tip: In this case, the two arcs are minor and major arcs of the circle. Since the measure of the major arc is $360°$ – (measure of minor arc), the difference between the two arc measures is $360°$ – 2(measure of minor arc), and half of this difference is $180°$ – (measure of minor arc). In other words, the angle between the tangents is the supplement of the minor arc.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 7 at the end of this lesson.

6. Some students have made statements based on their experiences with this TI-Nspire file.
   a. Bryan says that if two secants intersect in the interior of a circle, then the measure of an angle between them is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle. Do you agree? Which of your answers lend support for or against Bryan's statement?

      Answer: Yes. The answers to questions 2b and 2d support Bryan's statement.

   b. Dajah says that if two secants intersect in the exterior of a circle, then the measure of the angle formed is one-half the difference between the two intercepted arcs. Do you agree? Which of your answers lend support for or against Dajah's statement?

      Answer: Yes. The answer to question 3b supports Dajah's statement.

   c. Michael agrees with Dajah, but he also thinks her statement is true for the case when there is a secant and a tangent, and also for the case when there are two tangents. Do you agree? Which of your answers lend support for or against Michael's statement?

      Answer: Yes. The answers to questions 3b, 4b, and 5b support Michael's statement.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 8 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- If the intersection point *P* of two lines lies inside a circle, then the measure of the angle formed by the two secants is equal to the average of the measures of the arcs intercepted by that angle and its corresponding vertical angle.
- If the intersection point *P* of two lines lies outside a circle, then the measure of the angle formed is equal to half of the difference of the measures of the arcs intercepted by that angle and its corresponding vertical angle, whether the lines are both secants, a secant and a tangent, or two tangents.

TI-Nspire Navigator

Note 1

Question 1, *Quick Poll:* Send the following *Open Response Quick Poll* to students: What is the measure of major arc *ADB* if $m\angle AOB = 75$?

Answer: 285°

Note 2

Question 2a, *Quick Poll:* Have students answer 2a through an *Open Response Quick Poll*.

Note 3

Question 2b, *Screen Capture* and *Quick Poll:* Take a *Screen Capture* of students' screens as they are moving *P* around in the circle and trying to answer 2b so that they can look at several screens at once to find a relationship that is always true.

Once students have determined the relationship, send them the following *Open Response Quick Poll:* If arc *AB* = 40° and arc *CD* = 60°, then what is $m\angle BPC$?

Answer: 130

Note 4

Question 2d, *Quick Poll:* Have students answer 2d through a *Yes/No Quick Poll*.

Note 5

Question 3b, *Screen Capture* and *Quick Poll:* Take a *Screen Capture* of students' screens as they are moving *P* around in the circle and trying to answer 3b so that they can look at several screens at once to find a relationship that is always true.

Once students have determined the relationship, send them the following *Open Response Quick Poll:* If arc $AB = 40°$ and arc $CD = 60°$, then what is $m\angle APB$?

Answer: 10

Note 6

Question 4b, *Quick Poll:* Have students answer 4b through a *Yes/No Quick Poll*. Once students have determined that the relationship still holds, send them the following *Open Response Quick Poll:* If arc $AB = 40°$ and $m\angle APB = 70°$, then what is the measure of arc *AUD*?

Answer: $180°$

Note 7

Question 5b, *Quick Poll:* Once students have determined that the relationship still holds, send them the following *Open Response Quick Poll:* If arc $AB = 160°$, then what is $m\angle APB$?

Answer: $m\angle APB = 20$

Note 8

Question 6, *Quick Poll:* Have students answer question 6 through an *Agree/Disagree Quick Poll* and then explain why they agree or disagree.