Math Objectives

- Rotate and reflect two-dimensional figures in a coordinate plane.
- Analyze the coordinates of numerous rotated and reflected figures to determine the rules of geometric transformations.
- Use abstract and quantitative reasoning to explore relationships among pre-image and image figures.

Vocabulary

- reflection
- rotation
- transformation

About the Lesson

- Students will reflect figures over the *x*- and *y*-axes. By studying what happens to the coordinates of the pre-image and image points, they will write rules for reflecting points over the axes. Students will then rotate figures 90°, 180°, and 270° counterclockwise about the origin, studying the coordinates of the pre-image and the image to write rules for rotations. In Problem 3, students can use the "grab and move" feature to explore other reflections and rotations.
- Before beginning this activity, students should be familiar with reflections and rotations outside the coordinate plane. They should already know the terms associated with transformations, such as pre-image, image, line of reflection, angle of rotation, and center of rotation.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
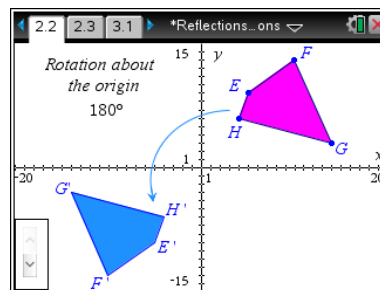
TI-Nspire™ Navigator™ System

- Send out the *Reflections_and_Rotations.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/ calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- Reflections_and_Rotations_ Student.pdf
- Reflections_and_Rotations_ Student.doc

TI-Nspire document

- Reflections_and_Rotations.t ns

Discussion Points and Possible Answers

Reflections in the coordinate plane can be over any line. This activity will begin by exploring reflections over the *x*- or *y*-axis. Similarly, a figure can be rotated clockwise or counterclockwise about any point in the coordinate plane, but this activity will begin by focusing on rotations about the origin.

You may wish to have students explore "less restrictive" and more advanced reflections and rotations using the constructions found on pages 3.1, 3.2, and 3.3 of the TI-Nspire document.

Problem 1 – Reflections

On page 1.2, $\triangle ABC$ is shown. Have students reflect this triangle across the *x*-axis. To reflect a figure, select the **Reflection** tool from the Transformation menu. Select the figure followed by the line of reflection.

Students can select the up arrow to verify that they reflected the triangle correctly. Selecting the down arrow will hide the reflection.
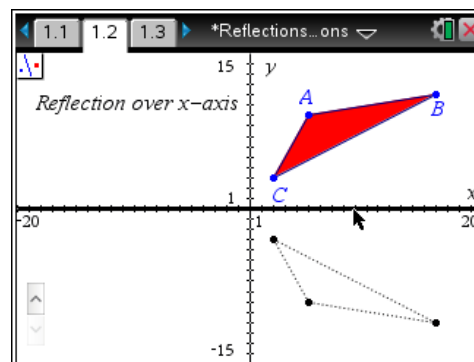
> **Tech Tip:** Use [tab] to select the up and down arrow slider. Then use the arrows on the handheld to change the slider for hiding/showing the reflection.

> **Tech Tip:** Select 🔧 > **Geometry > Transformation > Reflection**.
> Tap the shape to be reflected, and then tap the line to be reflected about.
> Note that in some cases, a student may need to back-out to the main Tools Menu 🔧 to see the desired menu option.

The coordinates of both sets of points can be displayed by choosing **MENU > Actions > Coordinates and Equations** and selecting each point. Have students record the coordinates on their worksheet and then sketch the triangles, labeling the image points with primes. (Labeling the points on screen may be accomplished by using the **Text** tool, also available from the Actions menu.)

Have students change $\triangle ABC$, dragging it by one of its vertices or a side. Ask students what they notice about any changes in the coordinates. (The *x*-values of pre-image and image points are equal; the *y*-values are opposites.)
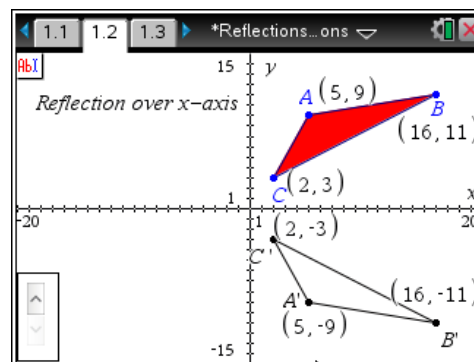
> **TI-Nspire Navigator Opportunity:** *Class Capture*
>
> **See Note 1 at the end of this lesson.**

Students will now reflect △*DEF* across the *y*-axis on page 1.3. Again, students can select the up arrow to verify they reflected the triangle correctly.

Once more, have students display and record the coordinates of the pre-image and image points, as well as label the image points with primes. Changing the coordinates of the pre-image points, ask students what they notice about the coordinates. (The *y*-values of pre-image and image points are equal; the *x*-values are opposites.)
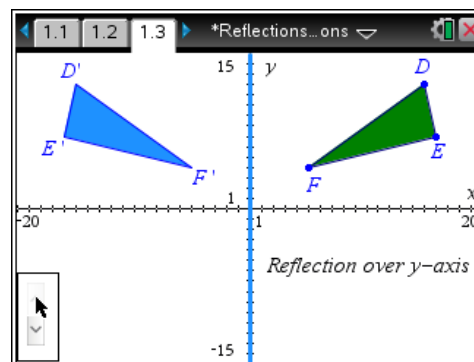
> **Teaching Tip:** Students have the opportunity to reason abstractly and quantitatively several times in this activity. They should pay attention to what is happening with the vertices of the polygons. Additionally, they should analyze the relationship between the numerous pre-image and image *x*-coordinates as well as the pre-image and image *y*-coordinates to determine a general rule for the given transformation.

Have students answer questions 1-4 on their worksheets.

1. What is the relationship between the *x*-coordinates of the pre-image and image when you reflect the figure over the *x*-axis?

    Answer: They are the same.

2. What is the relationship between the *y*-coordinates of the pre-image and image when you reflect the figure over the *x*-axis?

    Answer: They are the opposite.

3. What is the relationship between the *x*-coordinates of the pre-image and image when you reflect the figure over the *y*-axis?

    Answer: They are the opposite.

4. What is the relationship between the *y*-coordinates of the pre-image and image when you reflect the figure over the *y*-axis?

    Answer: They are the same.

Students should complete the rules on their worksheet. Introduce the following notation if necessary (the arrow is read as "goes to"):

reflect over *y*-axis: (*x, y*) → (−*x, y*)

reflect over *x*-axis: (*x, y*) → (*x, −y*)

Discuss with students why these rules make sense.

Make sure that they read the negative sign as "the opposite of" instead of "negative." The expression −*x* could be a positive coordinate if the *x*-coordinate of the pre-image is a negative number, e.g. (−4, −5) → (4, −5).

Problem 2 – Rotations

Next, students will explore rotations. On page 2.1, quadrilateral *ABCD* is drawn in Quadrant 1. Have students rotate the quadrilateral 90° counterclockwise about the origin.

To rotate the figure, choose the **Rotation** tool from the Transformation menu. Then select the figure to be rotated (*ABCD*), followed by the center of rotation (the origin), and the angle of rotation (**90°** as displayed in the text box). Students can use the up arrow to verify that they rotated the quadrilateral correctly. The down arrow will hide the quadrilateral.

Students should display and record the coordinates of both figures, and then sketch the pre-image and image on their worksheets, labeling the image points with primes. As before, students should spend some time dragging the pre-image around the coordinate plane. Ask how the coordinates are related. (The *x*- and *y*-values are exchanged, and the sign of the original *y*-coordinate is its opposite.)
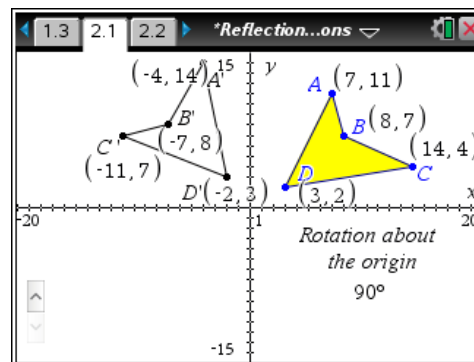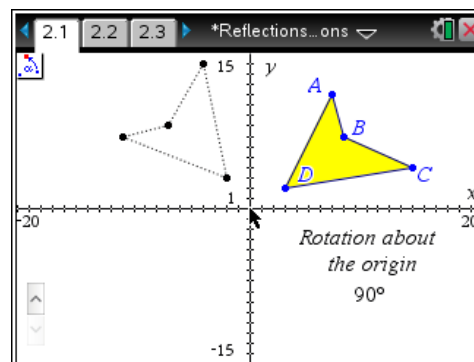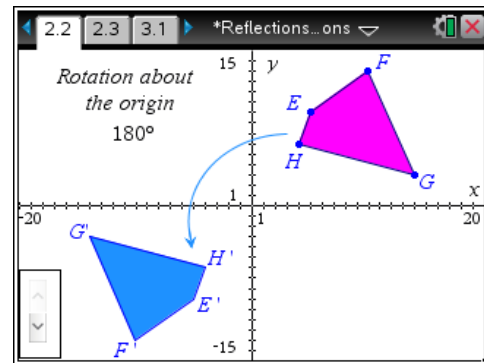
TI-Nspire Navigator Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

On page 2.2, students should repeat the above steps to explore 180° rotations about the origin, finding that the coordinates of the image and pre-image points are opposites.

The diagram on page 2.3 asks students to similarly explore 270° rotations counterclockwise about the origin. They should conclude that the *x*- and *y*-values are exchanged, and the sign of the original *x*-coordinate is its opposite.)

Students should now complete the rules on their worksheets:
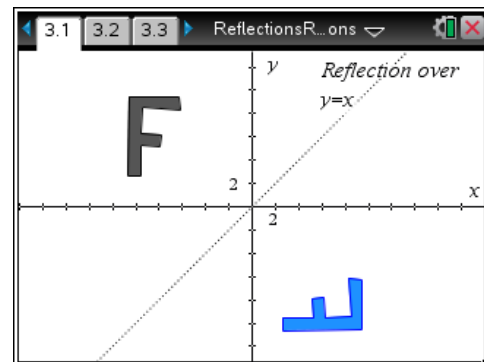
rotate 90° counterclockwise: $(x, y) \rightarrow (-y, x)$

rotate 180° counterclockwise: $(x, y) \rightarrow (-x, -y)$

rotate 270° counterclockwise: $(x, y) \rightarrow (y, -x)$
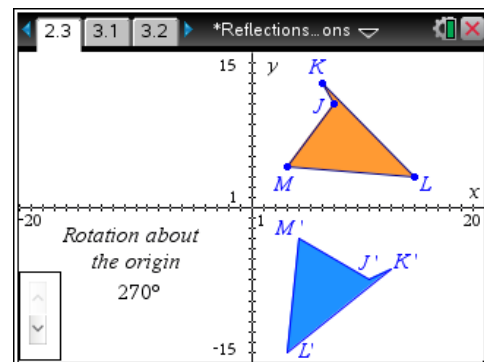
Problem 3 – More Reflections and Rotations

If time allows, encourage students to explore reflections over lines other than the axes using the diagram on page 3.1. The initial line given on page 3.1 is the line *y* = *x*. The line may be dynamically changed by dragging and/or rotating—and the image will update accordingly. Students can also grab and drag the pre-image.

Additionally, students can explore counterclockwise and clockwise rotations about the origin by different angle measures. On page 3.2, students can drag the topmost point on the circle around the circle to specify the angle of rotation. If students select the up arrow, they can rotate the figure clockwise. If students select the down arrow, they can rotate the figure counterclockwise.
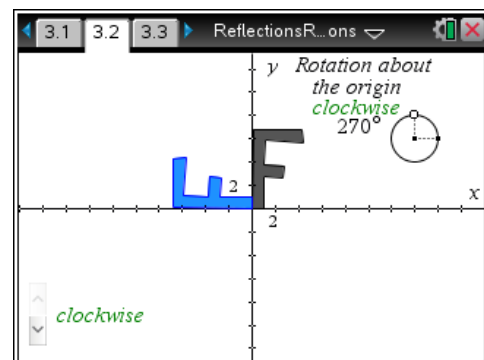
Page 3.3 takes the diagram from page 3.2 one step further by allowing students to also change the center of rotation, either by dragging the point directly, or by altering its coordinates displayed on the screen.

To alter the coordinates on the screen, double-click on the coordinate you want to change, type in the new value, and select enter .
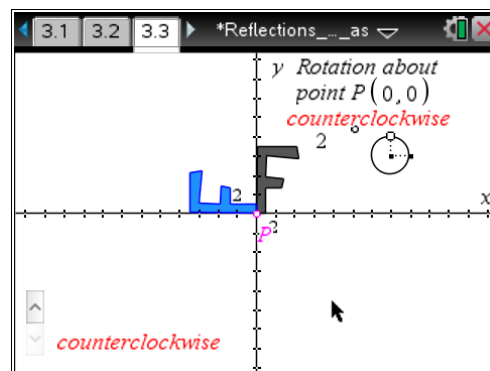
TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Class Capture*
This would be a good place to do a class capture to verify students are correctly reflecting the triangle.

Note 2
Problem 1, *Class Capture*
This would be a good place to do a class capture to verify students are correctly rotating the quadrilateral.