

# Cat and Mouse

## Teacher Answers



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7 8 9 **10** 11 12



TI-Nspire CAS



Investigation



Student



30min

## Aim

The aim of this investigation is to determine positive integer solutions for a game which is represented as a linear equation. Spreadsheets and graphs will be used to help identify possible solutions.

## Equipment

For this activity you will need:

- TI-Nspire CAS
- TI-Nspire CAS file – Cat and Mouse

## Problem Description

In the video game Cat and Mouse, you receive 4 points for each cat caught and 11 points for each mouse caught. What scores are impossible to get in this game? What is the largest impossible score?

Is there a pattern in the impossible scores?

The total score (S) can be represented by the equation:

$$S = 4 \times C + 11 \times M$$

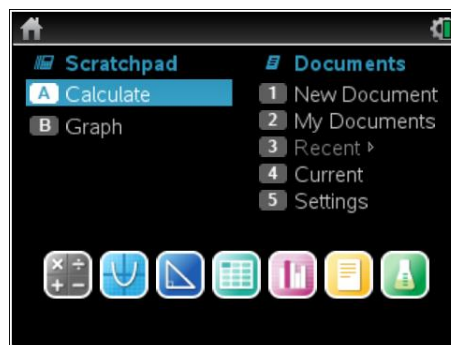
where “C” equals the number of cats and “M” equals the number of mice. The number of cats and/or mice can equal 0 (although not at the same time) and negative solutions are not allowed.

This equation with its integer solutions is an example of a Diophantine equation named after the 3<sup>rd</sup> century mathematician Diophantus of Alexandria who made a study of such equations. He was one of the first mathematicians to introduce symbolism into algebra. Diophantine equations may have a range of possible solutions from none to infinitely many integer solutions.

## Setting up the calculations

This activity requires access to the “Cat and Mouse” TI-Nspire document. This document should be loaded on your device before proceeding.

Once the document is on your handheld, press **home** and select **My Documents**. Locate the “Cat and Mouse” document and press **enter** to open.



### Determining if a score is possible with a spreadsheet

Navigate to screen 1.1 (shown opposite). Input a possible **score** in cell **A1** such as 30, then scroll down the **cats** or **mice** column until you find a pair of positive integer solutions. In the shaded row, 2 cats and 2 mice result in a total score of 30.

$$(2 \times 4 + 2 \times 11 = 30)$$

	score	cats	mice
1	30	0	30/11
2	30	1	26/11
3	30	2	2
4	30	3	18/11
5	30	4	14/11
6	30	5	10/11

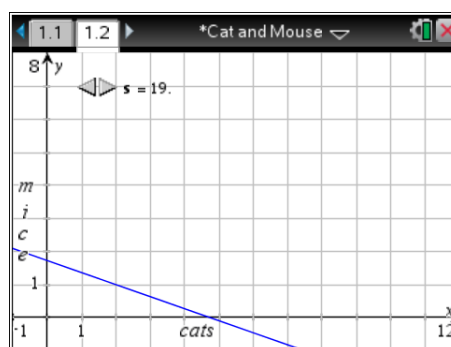
### Determining if a score is impossible

Input a score in cell **A1**, for example 13. Notice as the **cats** column increases from 0 to 4, the **mice** column decreases (as fractions) until a negative fraction is obtained. This indicates that a score of 13 is impossible to achieve.

	score	cats	mice
1	13	0	13/11
2	13	1	9/11
3	13	2	5/11
4	13	3	1/11
5	13	4	-3/11
6	13	5	-7/11

### Determining if a score is possible using a graph

Navigate to the screen 1.2 shown opposite. The slider in the top left hand corner indicates a score of **19**. Notice the graph passes through the point (2, 1). This indicates that the point (2, 1) is a solution. By using the slider, integer solutions can be found very quickly. Move the cursor to hover over the slider ◀▶ and click to increase or decrease the score.



Note: If you are using the TI-Nspire software, you can switch to the computer view to improve the accuracy of locating the intersection point of the line with the grid. If in doubt, verify the integer solutions with the spreadsheet on page 1.1.

## Questions

1. How many points do you receive if you catch 8 cats and 5 mice?

$$\begin{aligned} S &= 4 \times C + 11 \times M \\ &= 4 \times 8 + 11 \times 5 \\ &= 87 \text{ points} \end{aligned}$$

2. Gary obtained a score of 23. How many cats and mice did he catch?

$$\begin{aligned} S &= 4 \times C + 11 \times M \\ 23 &= 4 \times C + 11 \times M \\ 23 &= 4 \times 3 + 11 \times 1 \\ &3 \text{ cats and } 1 \text{ mouse} \end{aligned}$$

3. Which scores in the list below are impossible to get?

6, 12, 13, 15, 17, 21, 22

6, 12, 13, 15, 17, 21, 22

Scores of 6, 13, 17 and 21 are impossible to get.

4. Amanda obtained a score of 44. List two possibilities that result in a score of 44.

$T = 4 \times C + 11 \times M$  Since  $44 = 11 \times 4$ , then 11 cats or 4 mice will satisfy the equation.

Catching 11 cats and no mice equals a score of 44.

Likewise catching 4 mice and no cats results in a score of 44.

5. Adam caught four animals in total. What possible scores could he receive?

The results for four animals are:

Cats	Mice	Score
0	4	44
1	3	37
2	2	30
3	1	23
4	0	16

6. Which scores from 1 to 40 inclusive are impossible to get? From your results determine the largest impossible score.

The shaded scores are impossible to get.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,  
23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40

29 is the largest impossible score.

7. Once you know the largest impossible score, is it possible to obtain every score above this score? Investigate the next ten consecutive scores immediately after the largest impossible score. Comment on your findings. What do you notice?

The largest impossible score is 29. As soon as you have four scores in a row, (eg 30, 31, 32, 33) you can get any score by adding on 4. After 29, you can get every possible score.

8. One method of finding the highest impossible score is shown below.

$$\text{Highest impossible score} = \frac{xy - (x + y)}{\text{HCF}(x, y)}$$

where  $x$  and  $y$  are the number of points awarded and  $\text{HCF}(x, y)$  is the highest common factor of  $x$  and  $y$ .

For example in the Cat and Mouse game,

$$\text{Highest impossible score} = \frac{4 \times 11 - (4 + 11)}{1} = 29$$

What would be the highest impossible score for the following equations?

(i) Score =  $7 \times C + 11 \times M$

Highest impossible score

$$= \frac{7 \times 11 - (7 + 11)}{1} = 59$$

(ii) Score =  $9 \times C + 13 \times M$

Highest impossible score

$$= \frac{9 \times 13 - (9 + 13)}{1} = 92$$

(iii) Score =  $9 \times C + 15 \times M$

Highest impossible score

$$= \frac{9 \times 15 - (9 + 15)}{3} = 37$$

(iv) Score =  $15 \times C + 20 \times M$

Highest impossible score

$$= \frac{15 \times 20 - (15 + 20)}{5} = 53$$