**Concepts**

In the polar coordinate system, we select a point in the plane, called the pole, or origin, and label this point $O$. Next, draw a ray, or half line, starting at $O$, called the polar axis, usually drawn horizontally to the right. The polar axis corresponds to the positive $x$-axis in Rectangular, or Cartesian, coordinates.

Suppose $P$ is a point in the plane. Let $r$ be the distance from $O$ to $P$ and let $\theta$ be the angle (in radians) between the polar axis and the line $OP$. The point $P$ can be represented by the ordered pair $(r, \theta)$, and $r$ and $\theta$ are called polar coordinates of $P$. We usually use the convention that the angle is positive if it is measured counterclockwise from the polar axis and negative if it is measured in the clockwise direction. Note that if $P = O$, then $r = 0$. Therefore, $(0, \theta)$ represents the pole for any value of the angle $\theta$.

We can also interpret polar coordinates if the value of $r$ is negative. The points $(-r, \theta)$ and $(r, \theta)$ lie on the same line through the origin $O$ and both points are the same distance $|r|$ from $O$. However, they lie on opposite sides of the origin.

If $r > 0$, then the point $(r, \theta)$ lies in the same quadrant as the angle $\theta$. If $r < 0$, then the point $(r, \theta)$ lies in the quadrant on the opposite side of the pole. Any point in the plane can be represented in infinitely many ways using polar coordinates. For example, $(-r, \theta)$ and $(r, \theta + \pi)$ represent the same point in the plane.

The graph of a polar equation $r = f(\theta)$, or more generally, $F(r, \theta) = 0$, consists of all points $P$ such that there is at least one set of polar coordinates $(r, \theta)$ that satisfy the equation.

**Course and Exam Description Unit**

9.7: Defining Polar Coordinates and Differentiating in Polar Form

**Calculator Files**

Polar_Graphs.tns

# Exploring Polar Graphs

**MATH NSPIRED**

## Using the Document

This tns file is used to practice plotting and visualizing points in the plane represented by the polar coordinates $(r, \theta)$. The polar grid lines with common angles are displayed. Pages 2 and 3 are used to plot polar functions of the form $r = f(\theta)$.

### Page 1.1

| | |
|---|---|
| **Polar Coordinates and Graphs** <br><br> **(using polar grid lines)** <br><br> _____ <br><br> This file is used to practice plotting points represented by the polar coordinates (r, θ), and to construct graphs of polar equations in the form r = f(θ). | This introductory page explains the purpose of this tns file. The user can visualize points in the plane represented by polar coordinates, and plot polar functions of the form $r = f(\theta)$. |

### Page 1.2

| | |
|---|---|
| r = 3. <br><br> θ = $\dfrac{\pi}{6}$ <br><br> θ = 30° | This page is used to plot the points whose polar coordinates are of the form $(r, \theta)$. The values for $r$ and $\theta$ are set using the clickers on the left side of the screen. Note that the value of $\theta$ is given in radians and degrees. The point is displayed on the polar grid and a ray is drawn from the pole to the point. |

### Page 2.1

| | |
|---|---|
| On page 2.2, the user can define a polar function **r1(θ)**. <br><br> The initial polar function graphed on page 2.2 is the spiral **r1(θ)= θ**. | This page describes the method for graphing a polar function of the form $r = f(\theta)$, on page 2.2. The initial polar function that is graphed on page 2.2 is the spiral $r = \theta$. |

Page 2.2
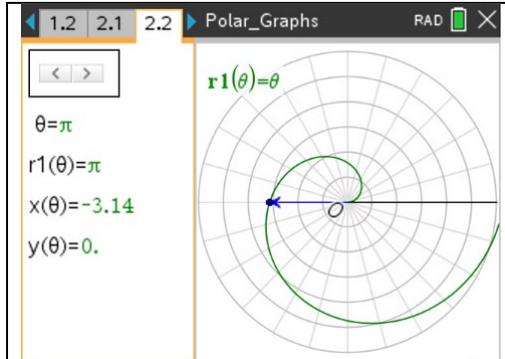
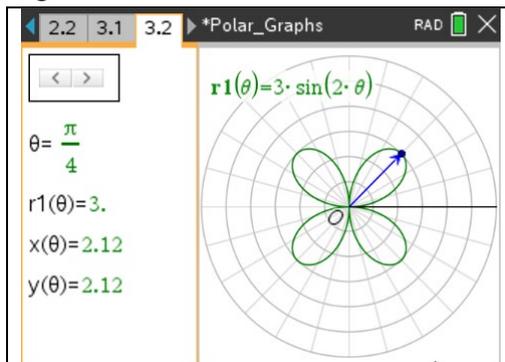| | |
|---|---|
|  | The graph of the polar function defined in the calculator function $r1$ is shown on the polar grid, in green, for $0 \leq \theta \leq 2\pi$. Use the clicker on the left screen to change the value of $\theta$. The corresponding point is shown on the graph, and an arrow is drawn from the origin to the point. As you click through values of $\theta$, the corresponding point moves along the curve. The value of $\theta$ is given in radians. The value of $r1$, and the $x$- and $y$-coordinates of the point are also shown on the left screen. |

Page 3.1

| | |
|---|---|
|  | This page describes the method for graphing a polar function of the form $r = f(\theta)$, on page 3.2. The initial polar function that is graphed on page 3.2 is $r = 3\sin(2\theta)$, a four-leaved rose. |

Page 3.2

| | |
|---|---|
|  | The graph of the polar function defined in the calculator function $r1$ is shown on the polar grid, in green, for $0 \leq \theta \leq 2\pi$. Use the clicker on the left screen to change the value of $\theta$. The corresponding point is shown on the graph, and an arrow is drawn from the origin to the point. As you click through values of $\theta$, the corresponding point moves along the curve. The value of $\theta$ is given in radians, and the value of $r1$, and the $x$- and $y$-coordinates of the point are also shown on the left screen. |

**Suggested Applications and Extensions**

Use Page 1.2 to help answer the following questions.

1. Plot the point whose polar coordinates are given. Find two other pairs of polar coordinates for this point, one with $r > 0$ and one with $r < 0$.

   (a) $\left(2, \dfrac{\pi}{6}\right)$

   (b) $(-1, \pi)$

   (c) $\left(2, -\dfrac{\pi}{6}\right)$

   (d) $\left(3, \dfrac{3\pi}{4}\right)$

   (e) $\left(4, -\dfrac{5\pi}{6}\right)$

   (f) $\left(-2, -\dfrac{4\pi}{3}\right)$

2. Plot the point whose polar coordinates are given. Find the Cartesian coordinates of the point.

   (a) $\left(3, \dfrac{\pi}{2}\right)$

   (b) $(-2, 3\pi)$

   (c) $\left(-4, \dfrac{\pi}{6}\right)$

   (d) $\left(1, \dfrac{3\pi}{4}\right)$

   (e) $\left(-2, \dfrac{7\pi}{4}\right)$

   (f) $(-1, -\pi)$

Use page 2.2 or 3.2 to sketch the graph of $r$ as a function of $\theta$. Describe the shape of the graph.

1. $r = -3\cos\theta$
2. $r = 1 + \sin\theta$
3. $r = 1 - \cos\theta$
4. $r = \theta^2$
5. $r = 2\cos(3\theta)$
6. $r = \sin\left(\dfrac{\theta}{3}\right)$     What values of $\theta$ produce a complete polar graph?
7. $r = 2 + 3\cos\theta$
8. $r = 1 + 2\sin\left(\dfrac{\theta}{2}\right), \quad 0 \le \theta \le 4\pi$
9. $r = e^{\sin\theta} - 2\cos(4\theta)$
10. $r = \sqrt{1 - 0.95\sin^2\theta}$