Factor Game

Teacher Notes and Answers

7 8 9 10 11 12

TI-Nspire Activity Student 120 min

Instructions – How to Play

The factor game involves two players. Player one starts by selecting a number between 1 and 49 inclusive. The selected number is added to player one's score, provided the number has at least one *proper factor*[1] remaining on the board. The selected number is immediately removed from the board. Player two automatically scores the sum of all the remaining proper factors, these factors are then removed from the board. It is now player two's turn to select a number.

Player two selects a number from those remaining on the board. Player two scores this number, provided at least one proper factor remains. The selected number is immediately removed from the board. Player one scores the sum of all the remaining proper factors which are then removed from the board.

If any player selects a number that does not have any proper factors remaining on the board, the selection is deemed invalid. When a player selects and invalid number they score zero points for that turn, so too their opponent, however play is then transferred to the opponent.

The game ends when neither player can make a valid selection.

Example: **Note: A PowerPoint show is included to provide a visual to help students learn how to play. Starting a game with a student in front of the class is also a good introduction.**

Player 1: The number 44 is selected. Proper factors of 44 are: {1, 2, 4, 11, 22}. As all of these numbers are currently on the board the selection is valid. The number 44 is immediately removed leaving player two to score: 1 + 2 + 4 + 11 + 22 = 40 points. The numbers: 1, 2, 4, 11, 22 and 44 are now all removed from play. It is now player two's turn to select a number.

Player 2: The number 33 is selected. Proper factors of 33 are: {1, 3, 11}, however only the number 3 remains on the board. As a proper factor is still in play the number 33 is valid so player two scores 33 points, taking their total to: 40 + 33 = 73 points. Player one scores the sum of the remaining proper factors: 3 bringing their total to: 44 + 3 = 47 points. The numbers now removed from the board include: {1, 2, 3, 4, 11, 22, 33, 44}. It is now player one's turn again.

Calculator & Board

Use counters and the board on the next page. A TI-Nspire file is also available called: "Factor Game Two Player". Load this file onto your calculator and run the FactorGame program from the [VAR] list on page 1.2. Even if you are using the calculator, it is worth tracking pieces on the board so that you are more conscious of the factors as they are identified.

[1] Proper Factor Example: Factors of 6: {1, 2, 3, 6}. Proper factors of 6: {1, 2, 3}. The original number is ignored when referring to proper factors. In some cases unity (1) is also ignored. For the purposes of this game, the number 1 is included as a proper factor.

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1	**2**	**3**	**4**	**5**	**6**	**7**
8	**9**	**10**	**11**	**12**	**13**	**14**
15	**16**	**17**	**18**	**19**	**20**	**21**
22	**23**	**24**	**25**	**26**	**27**	**28**
29	**30**	**31**	**32**	**33**	**34**	**35**
36	**37**	**38**	**39**	**40**	**41**	**42**
43	**44**	**45**	**46**	**47**	**48**	**49**

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Question & Discussion Points

Question: 1.

Which number would be better to start with: 46 or 39? Justify your answer and include supporting calculations.

Player 1 selects 46. Score: 46

Player 2 gets sum of factors: Score: 1 + 2 + 23 = 26

Player 1 leads by 20 points

Alternative:

Player 1 selects 39. Score: 39

Player 2 gets sum of factors: Score: 1 + 3 + 13 = 17

Player 1 leads by 22 points

So even though 46 is a higher score, selecting 39 provides a bigger lead, which is very important as the game progresses.

Teacher Notes:

The above reasoning only considers the first selection. When the number 39 is selected 13 is also removed from the board as a factor. (See above). This makes the number 26 more desirable as the only remaining factor is 2 therefore providing a scoring advantage of 24 points for whoever selects it. When the number 46 is selected 23 is removed, but 23 is not a factor of any other number on the board and therefore does not impact future selections. The smaller factors for both 26 and 39 are relatively insignificant.

Question: 2.

Which number is the best starting number? Justify your answer and include supporting calculations.

Player 1 selects 47. (The largest prime number, only be available for the very first player!)

Turn	Player	Selection	Available Proper Factors	Player 1 Progressive Score	Player 2 Progressive Score
1	1	47	1	47	1
2	2	49	7	47 + 7 = 54	1 + 49 = 50

Player 1 selects 49.

Turn	Player	Selection	Available Proper Factors	Player 1 Progressive Score	Player 2 Progressive Score
1	1	49	1,7	49	1 + 7 = 8
2	2	35	5	49 + 5 = 54	8 + 35 = 43

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If player 1 selects 47 to start the game, by the end of the second turn (assuming player 2 makes a logical choice), player 1 will have a lead of just 4 points.

If player 1 selected 49 to start the game, by the end of the second turn, player 1 will have a lead of 11 points.

So, whilst 47 points will score the best initial score, it leaves 49 in play… it is better to pick 49 first. Of course this argument only considers the first two plays; all plays until the game is exhausted would need to be considered to truly identify the best start.

Question: 3.

Which number, if any, would provide a higher score for your opponent if selected first?

Player 1 selects 12. Player 2 scores: 1 + 2 + 3 + 4 + 6 = 16 points.

Other numbers include: 12, 18, 20, 24, 30, 36, 40, 42 and 48.

These numbers are also called 'abundant' numbers.

Question: 4.

If the number 49 is selected first, what numbers will never be removed from the board?

All prime numbers above 24: {29, 31, 37, 41, 43, 47}. These numbers will no longer be valid as '1' has already been removed therefore a player cannot select them. These numbers will not appear as a 'factor' of a number since 29 x 2 > 49. Prime numbers less than 24 may still be removed as factors of larger numbers. For example if 46 is selected then 23 will be removed since 23 x 2 = 46.

Question: 5.

Will the combined score of player one and two be the same for every game?

The sum of all the numbers from 1 to 49 is equal to 1225. The combined score for any game is therefore 1225 less the sum of the numbers that remain at the end of the game. Now suppose 49 is selected and any of the following numbers: {10, 15, 20, 25, 30, 40, 45} are selected prior to 35. This would mean the number: 35 is no longer available for selection since 1, 5 and 7 would have been removed from the board.

Teacher Notes:

This is not sufficient evidence on its own to prove the overall sum will not be the same for every game, however it illustrates that the remaining numbers at the end of each game will most likely vary. Students could 'prove' the result by checking the overall sum for two games and show that different outcomes are possible!

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Question: 6.

Write down any strategies you developed whilst playing the game.

Answers will vary. Students should however establish that it is not the overall score that is most important rather the scoring advantage (difference between selection and sum of factors) that is most important followed by due consideration to the remaining numbers on the board.

Human vs Computer (Calculator)

Open up the TI-Nspire file: Factor Game Single Player. Start the game as per the previous document. For this game 'humans' go first, very polite. The aim of course is to beat the computer. Scores are done automatically.

Question: 7.

Play the factor game 5 times and record the scores for each game. Who won the most, computer or human?

Answers will vary

Teacher Notes

The purpose of this question is to see if students are engaging more successful strategies. Students may also engage reflective practices by observing the calculator's scores. Students may also use their results to justify their answer to Question 6.

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Question: 8.

Thinking just one move in advance; and selecting the best number for each move, what is the ideal sequence of numbers?

Player 1 Selection	Factor Score	Progressive Total	Player 2 Selection	Factor Score	Progressive Total	Selection Advantage
49		49		{1, 7}	8	41 (P1)
	5	54	**35**		43	30 (P2)
39		93		{3, 13}	59	23 (P1)
	2	95	**26**		85	24 (P2)
46		141		{23}	108	23 (P1)
	11	152	**33**		141	22 (P2)
45		197		{9, 15}	165	21 (P1)
	19	216	**38**		203	19 (P2)
44		260		{4, 22}	229	18 (P1)
	17	277	**34**		263	17 (P2)
28		305		{14}	277	14 (P1)
	6, 21	332	**42**		319	15 (P2)
30		362		{10}	329	20 (P1)
	8, 20	390	**40**		369	12 (P2)
32		422		{16}	385	16 (P1)
	12	434	**24**		409	12 (P2)
36		470		{18}	427	18 (P1)

Teacher Notes:

The total sum from this 'ideal' game is 897 leaving a total of 382 points on the board.

Numbers remaining: {25, 27, 29, 31, 37, 41, 43, 47, 48}

Interesting that the 'very' abundant number: 48 remains on the board as all its factors were progressively removed. In the second last move where the number 24 was selected, had player 2 selected 48 the overall result would have remained providing the same points advantage.

Player 2 = 48 points Player 1 = 24 + 12 = 36 points Point advantage: 12

The overall sum however would have been different, proving the result for question 5.

It is also interesting to note that the 'point advantage' does not decrease each turn suggesting that it may be strategically advantageous to select an invalid number at that point or consider point advantages beyond a single turn.

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Question: 9.

Using an example, show that 'thinking only about one move in advance' is not a sufficient strategy.

From the ideal game thinking one move in advance only when player 1 selects 39, this removes factors 3 and 13. This makes 26 a more attractive option with 2 is the only remaining factor. This discrepancy is observed in the reversal of values in the selection advantage.

Question: 10.

Alex played the computer and won by a total of 43 points. Is it possible to beat the computer by more than 43 points? Record you winning margin and the sequence of selections that lead to this margin. Note that when a game is over it is possible to use the cursor to move through all the selections for the game including progressive scores.

Answer: Maximum winning margin is yet to be determined by the author. If you manage to win be a larger margin, please email the selection sequence to: teacher-support@list.ti.com.

Teacher Notes:

This game clearly represents a significant exploration for students in year 7. It is an example of a task that can extend for the entire term as students make more discoveries with regards to strategies. Students need to find ways to document their exploration and resulting discoveries. Whilst it is possible to orchestrate a calculator vs calculator game, the calculator is only programmed to consider each move in isolation which has already been shown to be a flawed approach to the 'perfect' game.

Students could also explore other 'questions' such as: "Is the game fair?"

Teachers are encouraged to share their experiences exploring this game via the above email.

Of course there is another option for student exploration. The game presented in this exploration involves a board with the numbers 1 to 49. What if the board was change to a different size?

Author: P. Fox