Math Objectives

- Students will be able to use parametric equations to represent the height of a ball as a function of time as well as the path of a ball that has been thrown straight up.
- Students will be able to analyze parametric equations and use them to find information about the height of the ball and the time the ball is in the air.
- Students will apply the mathematics they know to solve problems arising in everyday life (CCSS Mathematical Practice).
- Students will use technology to visualize the results of varying assumptions (CCSS Mathematical Practice).
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).

Vocabulary

- parabola
- parametric equations
- quadratic equations

About the Lesson

- This lesson involves determining the height of a ball at a given time and determining the time at which the ball is at a certain height.
- As a result, students will:
    - Discover the similarities and differences between the height versus time graph and the graph of the path of the ball.
    - Understand that, while the height versus time graph is parabolic, the ball actually goes straight up and down.
    - Analyze how the solutions to an equation relate to the real-world situation modeled by the equation.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Live Presenter to provide assistance to students throughout the activity.
- Use Screen Capture to monitor students' progress.
- Use Quick Poll to assess students' understanding.

Parametric Ball Toss

_____

Determine how the path of a ball thrown up in the air is related to the graph of the height of the ball above ground as a function of time.
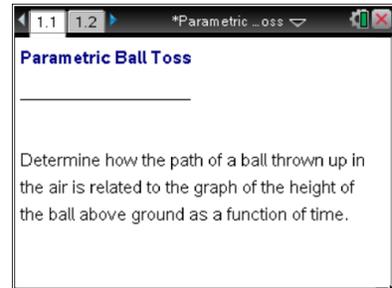
TI-Nspire™ Technology Skills:
- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:
- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing [ctrl] [G].

Lesson Files:
Student Activity
Parametric_Ball_Toss_Student.pdf
Parametric_Ball_Toss_Student.doc
Parametric_Ball_Toss_Create.doc

TI-Nspire document
Parametric_Ball_Toss.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** You might want to have students create their own .tns file for this activity. Directions to do so are provided in the additional Parametric_Ball_Toss_Create.doc file.

> **TI-Nspire Navigator Opportunity:** *Live Presenter and Screen Capture*
> **See Note 1 at the end of this lesson.**

Move to page 1.2.

At time $t = 0$, a ball is tossed straight up into the air. The graph on the right represents the height of the ball as a function of the time, t. The point to the left of the graph near the slider represents the ball in the air.
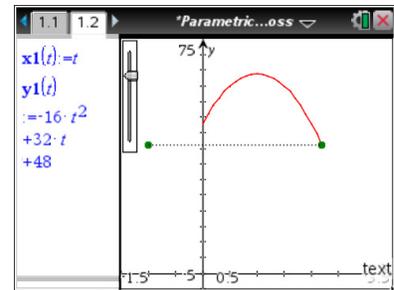
1. Drag the slider to observe the height of the ball, in feet, as a function of time, t, in seconds. Explain the shape of the function.

Sample Answers: Although the path of the ball is straight up and down, the graph of its height as a function of time is parabolic.

> **Teacher Tip:** You might want to point out to students that the graph is parabolic because the ball leaves the thrower's hand at a high speed, slows down until it reaches its maximum height, and then speeds up in the downward direction until it hits the ground.

2. The height of the ball as a function of time is modeled by the parametric equations $x_1(t) = t$ and $y_1(t) = -16t^2 + 32t + 48$.

    a. From what height was the ball thrown? How did you obtain this answer?

    Sample Answers: When the ball was thrown, $t = 0$. By substituting this into the equation, we see that $y_1(0) = -16(0)^2 + 32(0) + 48 = 48$. Thus, the ball was thrown from an initial height of 48 feet.

b. Where could the person throwing the ball have been standing? How do you know?

Sample Answers: The person must have been standing on the fourth floor of a building or on some kind of platform. We know this because, as we saw in part (a), the initial height of the ball was 48 feet.

> **Teacher Tip:** To help visualize this situation, you can remind students that one story of a building is approximately 10 feet.

3. As you drag the slider, observe the path of the ball to the left of the graph. Explain the shape of the path.

Sample Answers: Since the ball was thrown straight up, it travels up and down.

> **Tech Tip:** The path of the ball is obtained by plotting points whose $x2$ component remains fixed at -1 and whose $y2$ component is the same as $y1(t)$. See the associated Create document for more information on how to create these graphs.
>
> To eliminate the need to grab and drag the slider, you might want to animate the slider. To do so, click on the slider, and select ctrl **MENU > Animate**. To stop the animation, select ctrl **MENU > Stop Animate**.

You might want to utilize the scratchpad to help you answer some of the questions below. Press [📋] to access the scratchpad. By pressing [📋] again, you can toggle back and forth between the Calculator page and the Graphs page.

> **Tech Tip:** The menu features, Analyze Graph and Graph Trace, are only active when graphing in Function mode.

4. a. After how many seconds did the ball hit the ground? How do you know?

Sample Answers: The ball hit the ground three seconds after it was tossed.
The ball will hit the ground when its height is zero. We set $y_1(t) = 0$ and solve the equation $-16t^2 + 32t + 48 = 0$. We can solve this equation algebraically since the expression, $-16t^2 + 32t + 48$, can be easily factored. Alternatively, we can use Scratchpad to graph the function $y = -16x^2 + 32x + 48$ and find its positive zero.

b.   Explain a different method that you could have used to obtain your answer.

Sample Answers: Answers will vary depending upon the method students chose for part (b).

> **Teacher Tip:** You might want to specify that students solve the equation both graphically and algebraically. It is a good opportunity to have them practice using both methods of finding the zeros of the function.

> **Tech Tip:** To find the zero of the function, select **MENU > Analyze Graph > Zero**. Select the lower and upper bounds.
> Alternatively, select **MENU > Trace > Graph Trace**. As the cursor is moved, the zero is indicated.

> **Teacher Tip:** Now might be a good time to discuss why students should only choose the positive value of t.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

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5. a. What is the maximum height that the ball reached?

Answer: The ball reaches a maximum height of 64 feet. See part (c) for an explanation of how to determine this answer.

b.   How many seconds did it take the ball to reach that height?

Answer: It takes the ball one second to reach its maximum height of 64 feet. See part (c) for an explanation of how to determine this answer.

c. How did you obtain these answers?

Sample Answers: We can complete the square to obtain the vertex, or utilize the formula for finding the axis of symmetry.

To complete the square:

$$y_1(t) = -16t^2 + 32t + 48$$

$$= -16(t^2 - 2t) + 48$$

$$= -16(t^2 - 2t + 1) + 48 + 16$$

$$= -16(t-1)^2 + 64$$

To utilize the formula for the axis of symmetry:

$$t = \frac{-b}{2a} = \frac{-32}{2(-16)} = 1$$

Substitute $t = 1$ into the equation $y_1(t) = -16t^2 + 32t + 48$ to obtain $y_1(1) = 64$.

Using either method, since $x_1(t) = t$, the coordinates of the vertex are (1, 64), indicating that, in one second, the ball was 64 feet high.

Alternatively, we can use Scratchpad to graph the function $y = -16x^2 + 32x + 48$ and find the coordinates of its maximum value.

> **Tech Tip:** To find the maximum height of the function, select **MENU >
> Analyze Graph > Maximum**. Select the lower and upper bounds.
> Alternatively, select **MENU > Trace > Graph Trace**. As the cursor is
> moved, the maximum is indicated.

6. Two seconds after the ball was thrown up in the air, how many feet above the ground was it?

Answer: By substituting t = 2, we find that, two seconds after the ball was thrown, it was 48 feet high. We see that $y_1(2) = -16(2)^2 + 32(2) + 48 = 48$.

7. How many seconds did it take the ball to reach a height of 63 feet? How many seconds later does it reach that height again? Explain why the ball is able to reach a height of 63 feet twice.

Sample Answers: To find when the ball reached a height of 63 feet, we solve the equation, $-16t^2 + 32t + 48 = 63$. We obtain two answers, $t = 0.75$ and $t = 1.25$. The ball first reaches a height of 63 feet at 0.75 seconds, on its way up. It again reaches a height of 63 feet at 1.25 seconds (0.5 seconds later), on its way down.

8. Fill in the information in the table below:

Answer:

Time (sec)	0	0.5	1	1.5	2	2.5	3
Height (ft)	48	60	64	60	48	28	0

9. Using the information from the table above, determine the average speed of the ball during the indicated time interval:

   a. 0 to 0.5 seconds _____

   Sample Answers: We know that $rate = \dfrac{distance}{time}$. To determine how fast the ball was traveling, we must find how far it traveled and the elapsed time from 0 to 0.5 seconds. Thus,

   $r = \dfrac{60-48}{0.5-0} = \dfrac{12}{0.5} = 24$ ft/sec.

   b. 0.5 to 1 second _____

   Sample Answers: Utilizing the same method, $r = \dfrac{64-60}{1-0.5} = \dfrac{4}{0.5} = 8$ ft/sec.

   c. 1 to 1.5 seconds _____

   Sample Answers: Utilizing the same method, $r = \dfrac{60-64}{1.5-1} = \dfrac{-4}{0.5} = -8$ ft/sec.

d.   1.5 to 2 seconds _____

Sample Answers: Utilizing the same method, $r = \dfrac{48-60}{2-1.5} = \dfrac{-12}{0.5} = -24$ ft/sec.

10. What do the signs of your answers to question #9 tell you about the position of the ball?

Sample Answers: The positive value r = 24 ft/sec tells us that the ball is going up at a rate of 24 feet per second. The negative value r = -24 ft/sec tells us that the ball is going down at a rate of 24 feet per second.

> **Teacher Tip:** You might want to mention to students the difference between speed and velocity. In this case, the velocity is positive when the ball is going up and negative when the ball is going down. Speed is the magnitude of velocity. It is always positive or zero.

11. Does the ball slow down or speed up as it reaches its maximum height? Why?

Sample Answers: The ball slows down as it reaches its maximum height. During the first half-second after the ball was thrown, it traveled 24 feet. During the second half-second, it only traveled 8 feet. When the ball was first thrown, its initial velocity was a result of the force exerted by the person throwing the ball. As the ball traveled up, that force lessened as the force of gravity acted on the object.

12. How could you modify the equations for $x_1(t)$ and $y_1(t)$ to model a ball that was being thrown from a height of 4 feet? from ground level?

Sample Answers: Since t represents the time since the ball was thrown, $x_1(t)$ remains the same. Since $y_1(t)$ represents the height of the ball, that equation will change as we change the initial height. In the equation $y_1(t) = -16t^2 + 32t + 48,$ the ball was thrown from an initial height of 48 feet. If the initial height was changed to 4 feet, the new equation would be $y_1(t) = -16t^2 + 32t + 4.$ If the ball was thrown from ground level, the initial height would be zero, and the new equation would be $y_1(t) = -16t^2 + 32t + 0 = -16t^2 + 32t.$

13. How do these changes affect the time it takes for the ball to reach its maximum height? How do the changes affect the time it takes the ball to hit the ground? Explain why these answers make sense.

Sample Answers: Although the ball does not go as high, it takes the same amount of time to reach its maximum height. However, the ball does reach the ground sooner.

When the ball is thrown from an initial height of 4 feet, it reaches a maximum height of 20 feet in one second. It reaches the ground in slightly more than two seconds. When the ball is thrown from the ground, it reaches a maximum height of 16 feet in one second. It reaches the ground in exactly two seconds.

With all three initial heights, the balls reached their maximum heights in the same time, one second because the *x*-coordinate of the vertex of the parabola is not dependent upon the initial height of the ball, the value of *c* in the equation $y = ax^2 + bx + c$.

> **Teacher Tip:** You might want to point out to students that gravity always pulls with the same force regardless of the height from which the ball was thrown.

As the initial height decreases, the ball reaches the ground sooner. Since the ball doesn't go as high, it doesn't have as far to fall. Thus, it reaches the ground in less time.

Note that in all three cases, after the ball is in the air for two seconds, it returns to its initial height. In the first case, the initial height was 48 feet. Two seconds after the ball was tossed, it was again 48 feet off the ground. In the second case, the initial height was 4 feet. Two seconds after the ball was tossed, it was again 4 feet off the ground. In the last case, the ball started on the ground, with an initial height of 0 feet. Two seconds after the ball was tossed, it returned to the ground.

> **Teacher Tip:** Try to lead students to discover that the three graphs are simply vertical shifts of each other. In each case, the parabola is behaving the same way at the same time. The differences in the heights result from the vertical shifts of the parabola.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The differences between the height versus time graph and the graph of the path of the ball.
- That, while the height versus time graph is parabolic, the ball travels straight up and down.
- How to use the equations to obtain information about the height of the ball and the time it travels.

Assessment

You might want to give students different information for the height and time, and ask them to provide information about the height of the ball and the time the ball is in the air.

TI-Nspire Navigator

Note 1

Name of Feature: Live Presenter and Screen Capture

If students are creating their own .tns documents, you might want to have one of the students serve as the Live Presenter and demonstrate to the class how to navigate through the activity.

You also might want to leave Screen Capture running in the background, with a 30-second automatic refresh, without student names displayed, to enable you to monitor students' progress and make the necessary adjustments to your lesson.

Note 2

Name of Feature: Quick Poll

Use Quick Polls periodically throughout the activity to collect students' answers and assess their understanding. In particular, for question 4, be sure that students only select the positive value of *t* for the time it takes the ball to reach the ground.