Towering Mathematics

Teacher Answers

7 8 **9** **10** 11 12

TI-Nspire CAS | Investigation | Student | 90min

Aim:

To determine a relationship between the number of disks in the 'Tower of Hanoi' and the number of moves required to transfer the disks from one stack to another.

Equipment:

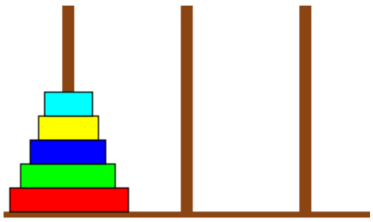
For this activity you will need:

- TI-Nspire CAS
- TI-Nspire CAS file (tns): HanoiV2

Problem Description:

The Tower of Hanoi problem, also called Tower of Brahma was invented by the French Mathematician, François Édouard Anatole Lucas in 1883. The objective is simple:

Move all the disks from one column to another.



The rules:

- Disks must always be stacked on a column (spike)
- Move only one disk at a time
- A large disk can not be placed on a small disk

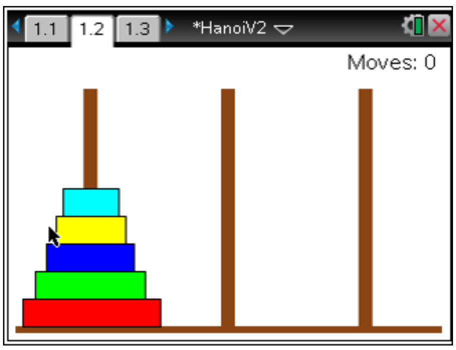
The aim of this investigation is to develop a rule relating the *number of disks* (x) and the *number of moves* (y) required to solve the problem.

Technology:

Open the file:

HanoiV2.tns

Navigate to the problem page (shown opposite).



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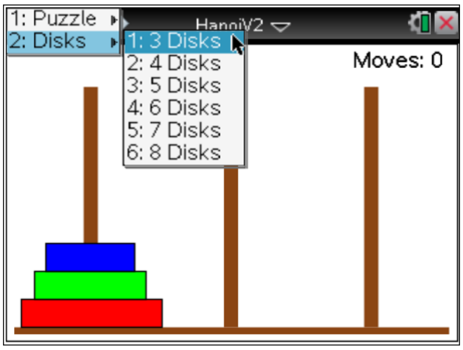
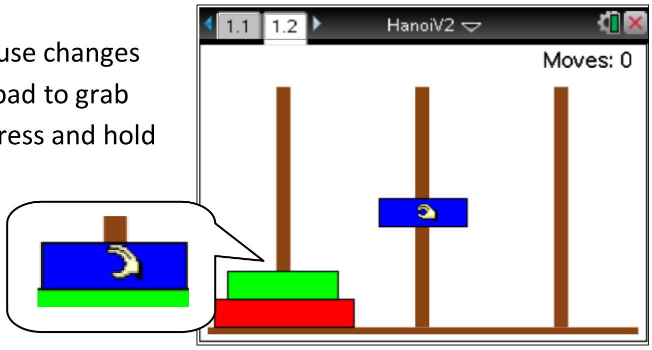
Setting the number of disks



Use the [**menu]** key and set the number of disks to three.

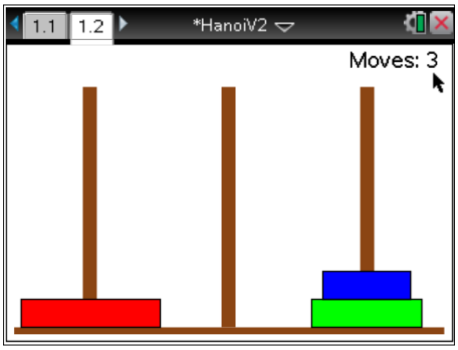
The single and two disk problems will be determined by reasoning rather than modelling.

Moving Disks



Move the mouse over the top disk, the mouse changes to an open hand. Press and hold the touchpad to grab the disk. Move the disk to a new column, press and hold the touchpad to release the disk.

Moves

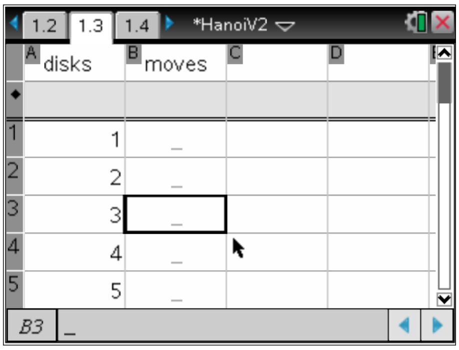


The number of moves is automatically recorded in the top right corner of the screen.

Once a move has been made, you can't take it back.

Solve the three disk puzzle and make a note of the number of moves.

Entering Data



Navigate to the spreadsheet on page 1.3

Enter the numbers 1 to 8 in column A.

Leave the first two entries in column B blank. These will be included by reasoning and logic later. Record the minimum number of moves used to solve the three disk problem in cell B3.

Questions

1. Use the puzzle to determine and record the minimum number of moves required to solve:

   a. The three disk problem. 7 moves

   b. The four disk problem. 15 moves

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If the number of moves increases by the same amount each time a disk is added the relationship is said to be linear.

2. If the relationship between disks and moves is linear, how many moves should it take to solve the five disk problem?
   Three disk problem = 7 moves
   Four disk problem = 15 moves
   Increased by 8 moves.

   If the relationship is linear then the five disk problem would take 23 moves. (15 + 8).

3. Use the puzzle to determine the minimum number of moves required to solve the five disk problem and hence determine if the relationship is linear or non-linear.
   Record your answer to the five disk problem in the spreadsheet.
   Five disk problem = 31 moves, therefore the relationship is non-linear.

4. Determine the number of moves required to solve the one and two disk problems. Explain how you determined these values.
   Record the number of moves in the spreadsheet.
   One disk problem = 1 move

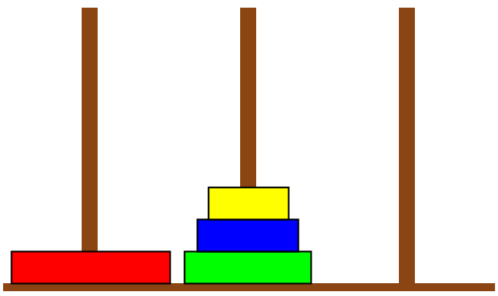
   Two disk problem = 3 moves

   Answers will vary: Draw a diagram, visualised problem or followed pattern represent typical answers.

5. Navigate to the graph page, with your data for the one, two ... and five disk problems in the spreadsheet you will see five points on the graph. Do the points on the graph form a straight line or a curve?
   The points form a curve, not a straight line.

6. The image shown opposite is a partial solution to the four disk problem. The large disk has yet to be moved.

   a. How many moves have taken place so far?
      The three disk problem has essentially been solved therefore 7 moves have taken place so far.



   b. Without referring to your previous data, how many moves are required to complete the puzzle?
      8 moves.

   c. Explain how you determined your answer to the previous question.
      The large disk needs to be moved (one move) then the three disk problem needs to be solved again (seven moves), total of 8 moves.

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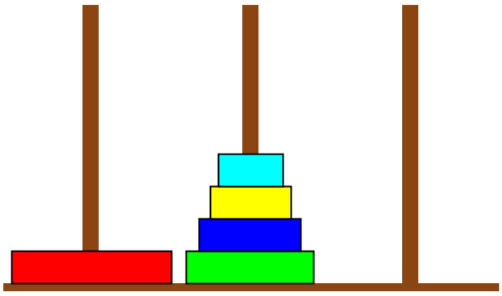
7. The image shown opposite is a partial solution to the five disk problem. The large disk has yet to be moved.



   a. How many moves have taken place so far?
   15 moves

   b. How many moves are required to complete the puzzle?
   15 + 1 = 16 moves

8. Use logic and your answers to the previous questions to predict the number of moves required to solve the six disk problem.
   31 moves for previous problem, therefore total moves for 6 disks: 31 + 1 + 31 = 63

9. Set the number of disks in the software to six and solve the problem; record your answer in the spreadsheet.

   Refer calculator spreadsheet.

10. Compare your prediction and practical solution in the previous two questions and explain any patterns you have observed relating the number of disks and moves.
    Double the previous answer and add one.

11. Predict the values for the seven and eight disk problems.
    Enter the predicted values in the spreadsheet.
    Seven disk problem: 127
    Eight disk problem: 255

Developing a Rule

The aim of this section is to develop a mathematical rule or formula for predicting the number of moves based on the number of disks in the problem.

12. Find the next 4 terms in the following sequence: 2, 4, 8, 16, ___, ___, ___, ___.

    Sequence: 2, 4, 8, 16, 32, 64, 128, 256

13. Compare the sequence in the previous question with the number of moves required to solve the one, two, three... and eight disk problems.

    Sequence from Tower of Hanoi: 1, 3, 7, 15, 31, 63, 127, 255

    The Tower of Hanoi moves are one less than the sequence in the previous question.

14. Navigate to the calculator page (1.5) and press the [**menu**] key, from the **number** menu select **factor**. Write the result for each of the following:

    a. Factor(4) Factor(4) = 2^2

    b. Factor(8) Factor(8) = 2^3

    c. Factor(16) Factor(16) = 2^4

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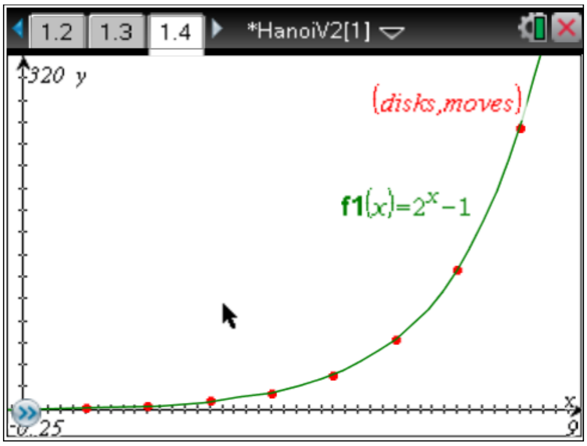
d. Factor(32) Factor(32) = 2^5

15. Use your answers to the previous three questions to formulate a rule relating the number of disks (x) and the number of moves (y) to solve the Tower of Hanoi problem.

Number of disks (x)	1	2	3	4	5	6	7	8
Number of moves (y)	1	3	7	15	31	63	127	255
Expression: 2^x	2	4	8	16	32	64	128	256
Rule: $y = 2^n - 1$	1	3	7	15	31	63	127	255

16. Check your formula from the previous question by drawing a graph on page (1.4). Note: You will need to change the graph type first. Press [**menu**] – **Graph Type** – **Function**.



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