Exploring Limits of a Sequence and Sum of a Series

MATH NSPIRED **TEACHER NOTES**

Background

The purpose of this calculator file is to numerically investigate the limit of a sequence a_n as $n \to \infty$, and the sum of the series $\sum_{k=1}^{\infty} a_k$. Sliders are used to quickly examine the value of a_n for large values of n, and to consider partial sums of the form $\sum_{k=1}^{n} a_k$, also for large values of n.

Course and Exam Description

Unit 10: Infinite Sequences and Series

Calculator File

Sequences&Series.tns

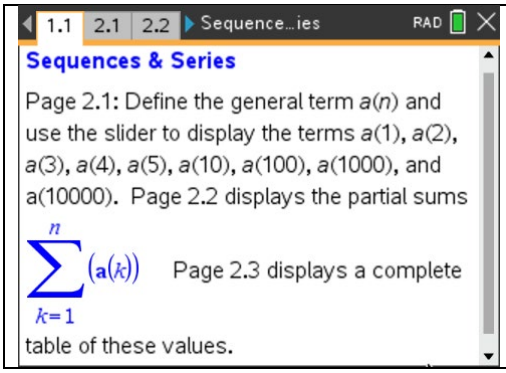
Using the Document

Sequences&Series.tns: This calculator file provides a technology tool for investigating the limit of an arbitrary sequence $\{a_n\}$ and whether an infinite series of the form $\sum_{k=1}^{\infty} a_k$ is convergent or divergent. A slider is used to display values of a_n and the partial sums $\sum_{k=1}^{n} a_k$ for various values of n. A table of these values is automatically computed and displayed in a Lists and Spreadsheet page.

The default sequence is $a_n = \frac{1}{n^3}$ and the corresponding series is $\sum_{k=1}^{\infty} \frac{1}{k^3}$.

The values for n used in this file are $n = 1, 2, 3, 4, 5, 10, 100, 1000, 10000$.

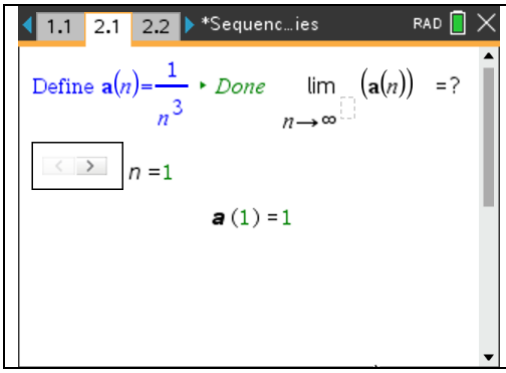
Page 1.1



This introductory screen provides information to help utilize this tns file. The general term of the series $\{a_n\}$ is defined on the calculator as (a function) $a(n)$. A slider is used to click through nine values of this sequence. On another calculator page, a similar slider is used to compute the partial sums $\sum_{k=1}^{n} a_k$ for these same nine values of n.

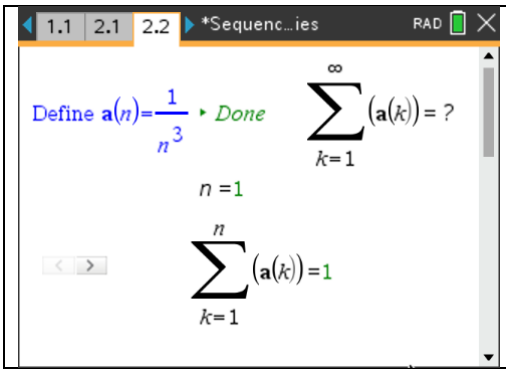
A complete table of values is given on page 2.3.

Page 2.1



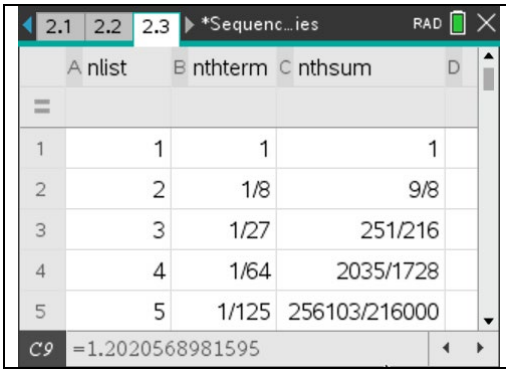
The sequence is defined in the Math Box at the top left portion of this Notes page. Remember to hit Enter after defining a new sequence. Use the slider for n to click, or scroll, through the preprogrammed values of n. The corresponding value for a_n is displayed in the middle of the screen. This numerical tool is designed to guess the limit $\lim_{n\to\infty} a_n$.

Page 2.2



This calculator page is used to investigate the partial sums of the series $\sum_{k=1}^{\infty} a_k$. The Math Box in the top left portion of this Notes page is used to define the terms of the sequence $\{a_n\}$. This Math Box and the similar one on Page 2.1 are linked; the user can define the sequence on either page. Use the slider for n (bottom left) to click, or scroll, through the preprogrammed values of n. The corresponding partial sums are displayed.

Page 2.3



This Lists and Spreadsheet page displays a complete table of values for n, a_n, and $\sum_{k=1}^{n} a_k$. Note that some of the values for a_n and $\sum_{k=1}^{n} a_k$ are presented in exact symbolic form. You might consider creating other columns in this Lists and Spreadsheet page with all computed values presented as decimal approximations.

Suggested Applications and Extensions

Find several values of each sequence. Use these values to conjecture if the sequence converges or diverges. If you think it converges, guess the limit.

1. $a_n = \dfrac{7 - 5n^2}{3 + 10n}$
2. $a_n = \left(\dfrac{1}{e}\right)^n$
3. $a_n = \dfrac{n}{e^n}$
4. $a_n = \dfrac{\ln n}{n}$
5. $a_n = \dfrac{n^n}{n!}$
6. $a_n = \dfrac{\cos n}{n}$
7. $a_n = \left(3 + \dfrac{3}{n}\right)^n$
8. $a_n = \dfrac{\sin(n\pi)}{3^n}$
9. $a_n = \sqrt[n]{2^n + 3^n}$
10. $a_n = \tan^{-1}\left(\dfrac{-n^2}{n^2 - 7}\right)$
11. $a_n = \ln(n) - \ln(n+1)$
12. $a_n = e^{1/\sqrt{n}}$

Find several partial sums for each series. Use these values to guess whether the series is convergent or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^2 + n}$
2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n^2 - n + 3}$

4. $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

5. $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{1}{n!}$

7. $\sum_{n=1}^{\infty} \cos n$

8. $\sum_{n=1}^{\infty} (-1)^{n-1} e^{3/n}$

9. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{2n^3 + n^2 - 7n + 5}$

10. $\sum_{n=1}^{\infty} \frac{\cos n}{n!}$

Extended Application Questions

1. Determine whether there is a relationship between series convergence and the terms of the corresponding sequence. Are there any general sequences $\{a_n\}$ such that the corresponding series $\sum_{n=1}^{\infty} a_n$ is guaranteed to converge? Diverge?
2. In those series that contain some terms that are positive and some terms that are negative, consider the series of the absolute value of each term, that is, $\sum_{n=1}^{\infty} |a_n|$. Is there a relationship between the convergence or divergence of $\sum_{n=1}^{\infty} |a_n|$ and the convergence or divergence of $\sum_{n=1}^{\infty} a_n$?