Solid of Revolution

Student Activity

7 8 9 10 11 **12**

TI-84CE | Activity | Student | 60 min

Introduction

A liquid flows into an irregular shaped vase at a constant rate, is it possible to describe the changing height of the fluid using mathematics? In this investigation you will collect data for the height of a liquid as it flows into a vase, produce mathematical equations to model these changes and rates, determine an equation for the volume of the liquid at any height *h*, and finally, produce a 3D model of the vase!

Equipment

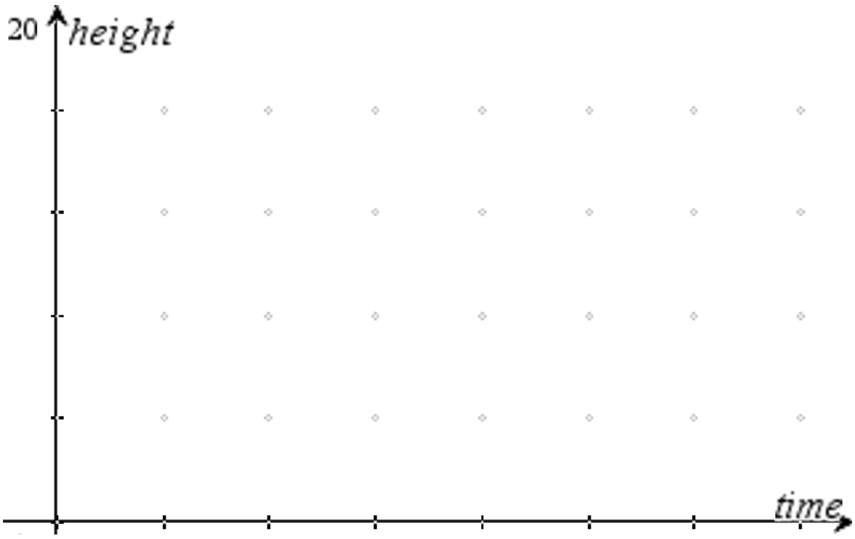
- Image of Vase
- YouTube video
- TI-84CE Calculator
- TI-Connect CE software
- Calculator to Computer Link cable

Filling the Vase

Question: 1.

Imagine the vase pictured above is being filled at a constant rate to a depth of 20cm (as shown). If this process takes 90 seconds, sketch a graph of the height versus time.

Answers will vary – key elements include starting at the origin, height increases most slowly at the start, gradually increasing as it passes through the narrow, middle section of the vase, then slowing again as the vase gets wider nearer the top. The graph should be a continuous smooth curve.



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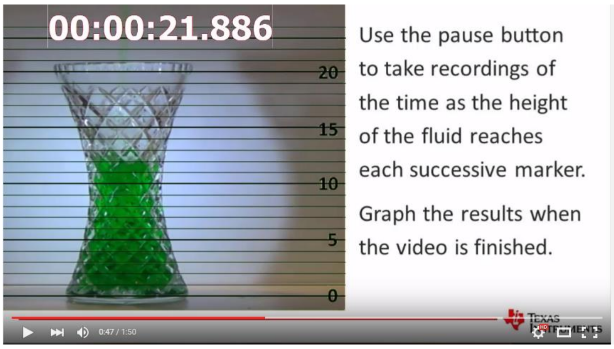
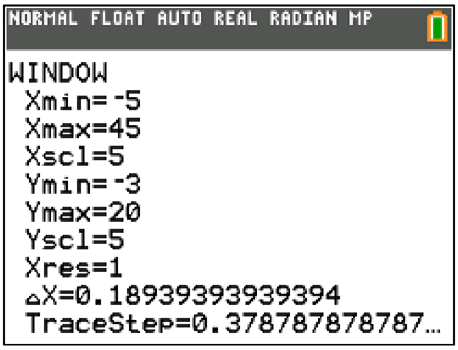
Use List 1 (L_1) to record the time and List 2 (L_2) for the height of the liquid as the vase in the video is filled at a constant rate[1]. Use the video speed and pause option on YouTube to slow and stop the video as the liquid reaches each graduation mark. Note that the splashing and crystal structure of the vase make some reading difficult; however they are a part of the real problem.



Related Rates and Solid of Revolution
http://bit.ly/JugFillingVase2

Graph your data as a scatter plot with TIME (L_1) on the independent axis and HEIGHT (L_2) on the dependent.

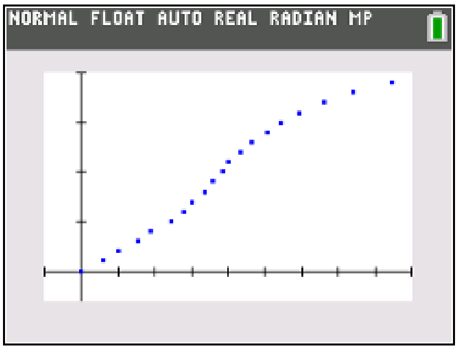
Approximate window settings are shown opposite.



Note: In some sections of the video it is difficult to see the exact height of the liquid in the vase. Reducing the speed of the video and using pause/rewind are useful tools to enesure the maximum accuracy possible. Some parallax error also exists, however, despite all of this the data is generally very good. The biggest discrepency in this activity relates to the thickness of the vase.

Question: 2.

Relate key features of the graph to the shape of the vase including a comparison of your sketch graph with the one obtained from the video data.



A typical data set is shown opposite. The key features are easy to see from the data:

- Starts at the origin
- Height changes slowly when vase is relatively empty (and wide)
- Height changes more rapidly when vase is narrow. Individual points on the graph serve as useful references and examples of 'average rate of change' would be useful inclusions.

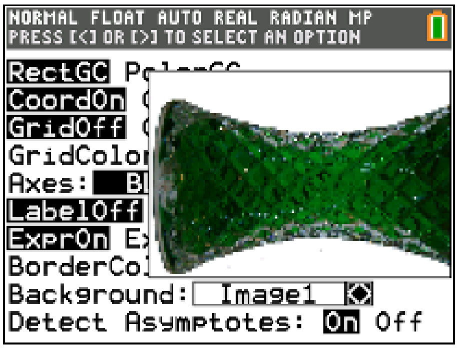
[1] Filled at a constant rate = The change in volume with respect to time is constant. This quantity can be computed using the amount of time it takes to fill the vase and knowing the volume of the vase: 800ml at 18cm depth.

Volume of the Vase

Note: Make sure your document settings have Float = 5 or 6. Due to the practical nature of the questions in this section, calculations need a high level of accuracy in order to model the shape of the vase.

Use the TI-Connect CE software[2] to drag and drop the image into your calculator. You can also use a calculator to calculator cable and the LINK option [2nd] [xtθn] on the calculator to send the image from one calculator to another.

Use the FORMAT menu to place the image of the vase in the background.



Two sets of window settings will be used repeatedly in this activity. To quickly recall previous window settings use [zoom] and move across to select Memory at the top of the screen. Current window settings can be saved using Zoom Sto and later recalled Zoom Rcl.

Turn off the Scatter Plot and adjust the window settings as follows:

XMin = -1 XMax = 20

YMin = -6.5 YMax = 6.5

These window settings produce a true aspect ratio and scale as it relates to the image of the vase.

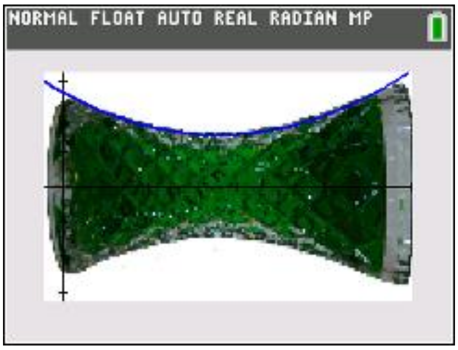
Use the [Trace] key to determine an approximate coordinate for the turning point of a parabola that will model the curvature of the vase.

Question: 3.

Determine the equation for a quadratic function that represents the curvature of the vase.

Important: Place your equation in Y_1

Student answers will vary. The graph shown is the parabola: $y = 0.03(x-9)^2 + 3$ A more accurate fit can be obtained but this generally results in more difficult numbers later in the investigation.



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[2] TI-84CE Connect Software (FREE) = MAC version: http://bit.ly/TI84CE_Link_MAC PC version: http://bit.ly/TI84CE_Link_PC

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Question: 4.

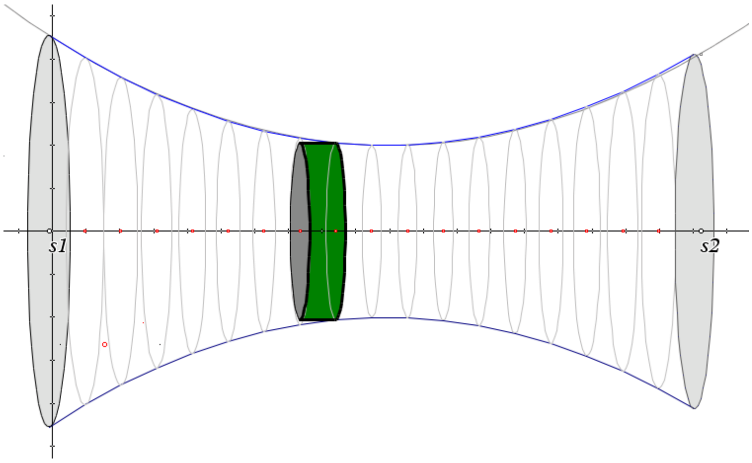
Explain why the vase is easier to model when tipped on its side.

Having the vase on its side means the curvature can be modelled by a function rather than a relation. ie: Many to One mapping (side) vs One to Many (upright)

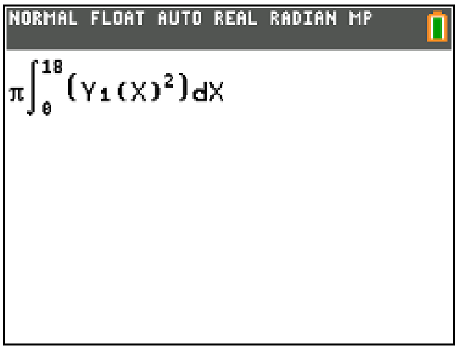
The theoretical volume of the vase can be determined by rotating the function around the x- axis.

$$V = \pi \int f(x)^2 \, dx$$

The actual volume of the vase is 800ml when filled to a depth of 18cm, the amount shown in the original picture.



The calculator can be used to determine the approximate volume of liquid in the vase using the syntax shown opposite.



Question: 5.

According to the calculator, what is the volume of the vase when filled to a depth of 18cm? Discuss the accuracy of the result including the potential cause for any discrepancies.

Answers will vary depending on the student's equation. $\pi\int\left(0.03(x-9)^2+3\right)^2 dx \approx 850ml$

Discrepancies include the small image (calculator screen) therefore reasonable to expect at least 10% error. There is also an assumption that the curvature of the vase is parabolic. A more significant error however occurs in the thickness of the vase. Example: $\pi\int\left(0.03(x-9)^2+2.8\right)^2 dx \approx 767ml$

this assumes the vase walls are approximately 2mm thick.

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Question: 6.

Use calculus to determine an expression for the volume of liquid (v) in the vase as a function of height (h).

Store this function in Y_2.

$$v(h) = \pi \int_0^h \left(0.03(x-9)^2+3\right)^2 dx$$

$$= \pi \int_0^h \left(0.0009(x-9)^4+0.18(x-9)^2+9\right) dx$$

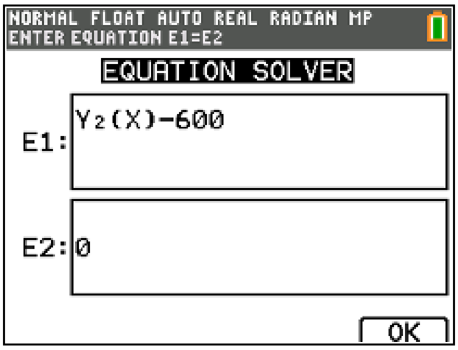
$$= \pi \left[\frac{0.0009}{5}(x-9)^5+\frac{0.18}{3}(x-9)^3+9x\right]_0^h$$

$$= \pi \left(\frac{0.0009}{5}(h-9)^5+0.06(h-9)^3+9h\right)+54.3688\pi$$

Question: 7.

Use the Numeric Solver on the calculator to determine an approximate value for the height of the liquid in the container when it contains 600ml.

Note: Use the function stored in Y_2 to 'paste' the equation into the solver, as shown opposite.



Answers will vary depending on student's equations.

Using the example equation: $h \approx 14.2$cm

Question: 8.

Given that the liquid is flowing into the vase at a constant rate, check your answer to question 7 using your data.

Given 800ml of liquid flows into the vase in approximately 42.3 seconds and that this rate is constant, then 600ml would have flowed in at 0.75 x42.3 = 31.7 seconds.

Using the data, the height of the liquid in the vase was approximately 16cm when t = 31 ~ 32 s.

Graphing the Height

The aim of this section is to determine a rule that describes the ***rate*** at which the height of the liquid is changing using a combination of the formula generated for the volume (based on the shape of the vase) and compare this to the data that was collected from the video.

Consider the following:

$\frac{dv}{dt} = constant$	This can be determined from information provided and collected already.
$v(t) = \int constant\ dt$	Remember that the initial volume of fluid was 0 ml.

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Question: 9.

Determine a rule for: $v(t)$

$$v(t) = \int 18.9dt$$
$$v(t) = 18.9t + c \qquad but \quad v(0) = 0$$
$$v(t) = 18.9t$$

Question: 10.

Using the answer to Question 6 and Question 9, determine a rule relating *time* and *height*.

Note: This rule is best expressed as $t(h)$.

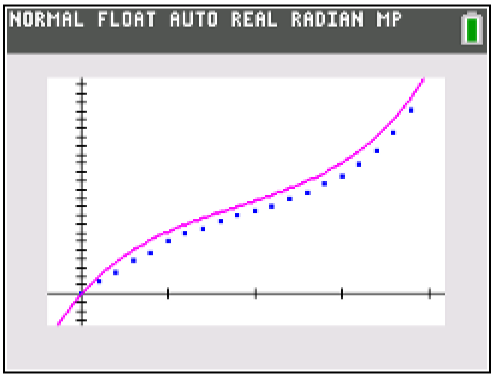
$$v = \pi\left(\frac{0.0009}{5}(h-9)^5 + 0.06(h-9)^3 + 9h\right) + 54.3688\pi \qquad and \qquad v = 18.9t$$
$$t = \frac{\pi}{18.9}\left(\frac{0.0009}{5}(h-9)^5 + 0.06(h-9)^3 + 9h + 54.3688\right)$$

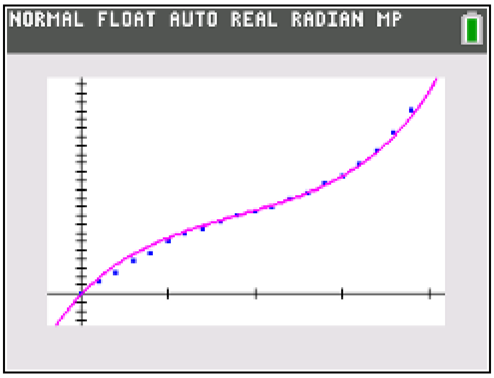
Question: 11.

Change the settings for the scatterplot to have *height* on the independent axis (x axis) and *time* on the dependent (y axis) and then graph the function from Question 10. Discuss how accurately the function matches the data.

Note: A variation of 0.01 in the original formula for the curvature of the vase makes a significant difference in the final relationship.

The accuracy of the graph depends on the data collected and the equation for the curvature of the vase. The error is largely attributable to the thickness of the glass in the vase. The two graphs below show what happens when the thickness of the vase is considered. The graph on the left is produced using a curvature equation: $y = 0.03(x-9)^2 + 3$; the graph on the right: $y = 0.03(x-9)^2 + 2.7$.





Note: Assuming the original function is entered in Y_1 and liquid is entering the vase at a rate of: 18.9ml/s, students can use: $\frac{\pi}{18.9}\int_0^x (y_1(x))^2 dx$

Warning: This graph is a VERY slow process as each point is plotted by first integrating the equation, such as:, $\frac{\pi}{18.9}\int_0^{0.1} (y_1(x))^2 dx$, $\frac{\pi}{18.9}\int_0^{0.2} (y_1(x))^2 dx$, $\frac{\pi}{18.9}\int_0^{0.3} (y_1(x))^2 dx$ ...

To speed this process up considerably, change the Xres setting in the Window menu to Xres = 5.

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