Systems of Equations

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will recognize a system of equations.
- Students will know the definition of a system of an equation.
- Students will determine solutions to a system of equations graphically.
- Students will verify solutions to a system algebraically.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).
- Students will analyze and solve pairs of simultaneous linear equations (CCSS).
  - Students will understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  - Students will solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations.

Vocabulary

- point of intersection
- parallel lines
- system of equations
- variables
- solution(s) to a system of equations

About the Lesson

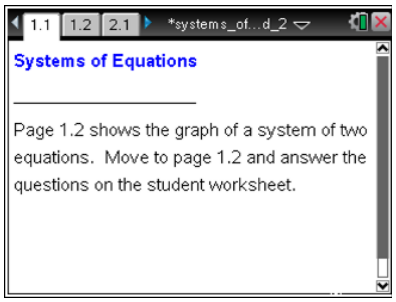
- This lesson involves definitions of a system of equations and solutions to a system of equations.
- As a result, students will:
  - Examine a graph of a system of two equations and two unknowns. They will be able to drag a point along the *x*-axis and as they do, corresponding points will move along the two graphs of the equations.
  - Recall that any point on a particular equation is a solution to that given equation. They should also notice that to solve a system of equations, they are looking for a solution that simultaneously solves both equations.

TI-Nspire™ Navigator™

- Send a File.
- Use Class Capture to examine patterns that emerge and formerly assess students' understanding.
- Use Live Presenter to demonstrate and provide a means for students to share their thinking.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Systems_of_Equations_Student.pdf
- Systems_of_Equations_Student.doc

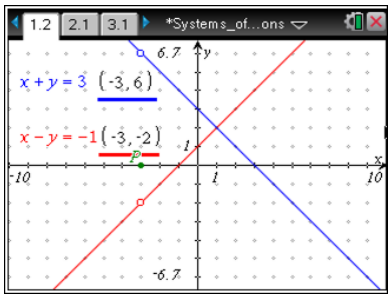
TI-Nspire document

- Systems_of_Equations.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl ☜ to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

Move to page 1.2.



A ***system of equations*** is a set of two or more equations with the same variables. In ***solving a system of equations***, your goal is to find values for these variables that satisfy all equations in the system simultaneously. For example, the system below consists of two equations and two variables. In this case, the variables are *x* and *y*.

$$\begin{cases} x - y = -1 \\ x + y = 3 \end{cases}$$

To solve this system, your goal would be to find values of *x* and *y* that simultaneously satisfy both equations. In Questions 1-3, we will refer to this system and work towards finding the solution.

1. On page 1.2 the graphs of both of these equations are shown. On each of these graphs, a point is labeled with its coordinates. There is also a point *P* on the *x*-axis that is not labeled. What do all three points have in common? What is different?

   Sample Answers: The *x*-coordinates of all three points are the same. The *y*-coordinates are all different.

   Teacher Tip: You may want to demonstrate how to drag and move point P. You might also want to encourage students to think about the coordinates of point P as they are not labeled.

   Tech Tip: To see the coordinates of a point, press on it twice. Of the pop-up options that appear, select "Coordinates."

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

2. Move point *P* so that $x = {}^{-}3$ in both sets of coordinates. The graph shows that the point $({}^{-}3, 6)$ is a solution to the equation $x + y = 3$. This can be shown algebraically. When you substitute the x- and y-coordinate values into the equation, the statement is true. That is, ${}^{-}3 + 6 = 3$. How can you tell that the point (-3, 6) is not a solution to the equation x – y = -1? Justify your answer graphically and algebraically.

Sample Answers: When you replace -3 for *x* and replace 6 for *y* in the equation $x - y = {}^{-}1$ you get ${}^{-}3 - 6 = {}^{-}1$. This is a false statement, because ${}^{-}9$ doesn't equal ${}^{-}1$. Thus, $({}^{-}3, 6)$ is **not** a solution to the equation $x - y = {}^{-}1$.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

3. Move point *P* until you find a point that will make both equations true at the same time. How do you know you have found the solution to the system graphically? How can you show that you have the simultaneous solution algebraically?

Sample answers: The simultaneous solution is the point $(1, 2)$. This can be seen graphically because this is a point on the graphs of both equations. Algebraically this can be seen because in the first equation you get 1 + 2 = 3 and in the second you get $1 - 2 = {}^{-}1$, which are both true statements.

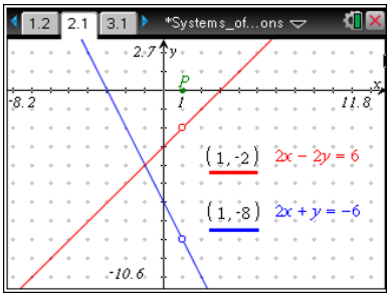
TI-Nspire Navigator Opportunity: *Class Capture:*
See Note 3 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Quick Poll:*
See Note 4 at the end of this lesson.

Move to page 2.1.

In Questions 4-6, we will use the graphical and algebraic methods discovered in the previous problems to explore the system below.

$$\begin{cases} 2x - 2y = 6 \\ 2x + y = {}^{-}6 \end{cases}$$



4. Which of the following statements is true and how do you know?

The point $(1, -8)$ is a solution to only one of the equations.
The point $(1, -8)$ is a solution to the system of equations.
The point $(1, -8)$ is a solution to none of the equations.

Sample answers: The point $(1, -8)$ is a solution to only one of the equations because it is only on the graph of $2x + y = -6$. In addition, algebraically $2(1) + -8 = -6$ is a true statement.

Teacher Tip: To increase student understanding that the solution makes the equation true, encourage them to verify their findings algebraically as well as graphically.
Mathematical Practice: Look for and make use of structure.
Mathematically proficient students should see that (1,-8) is one of many solutions to the complicated equation 2x + y= -6. They should notice when they move point p to 1 on the x-axis that it identifies the point on the line and satisfies the equation by making a true statement.

TI-Nspire Navigator Opportunity: *Quick Poll:*
See Note 5 at the end of this lesson.

5. Which of the following statements is true and how do you know?

The point $(-1, -4)$ is a solution to only one of the equations.
The point $(-1, -4)$ is a solution to the system of equations.
The point $(-1, -4)$ is a solution to none of the equations.

Sample answers: The point $(-1, -4)$ is a solution to the system of equations because it is on the graph of both equations. In addition, algebraically $2(-1) - 2(-4) = 6$ and $2(-1) + -4 = -6$ are true statements.

6. Which of the following statements is true and how do you know?

The point $(-4, 5)$ is a solution to only one of the equations.
The point $(-4, 5)$ is a solution to the system of equations.
The point $(-4, 5)$ is a solution to none of the equations.

Sample answers: The point $(-4, 5)$ is a solution to none of the equations because it is not on either graph. In addition, algebraically $2(-4) - 2(5) = 6$ and $2(-4) + 5 = -6$ are false statements.

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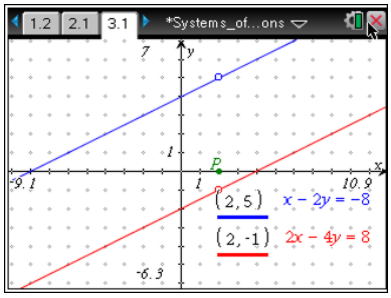


In Question 7, we will refer to the system below and work towards finding a simultaneous solution.

$$\begin{cases} x - 2y = -8 \\ 2x - 4y = 8 \end{cases}$$

7. How is this system different than the previous systems?

Sample answers: The graphs of these equations do not intersect. The lines are parallel.

8. What is the solution to this system?

Sample answers: There is no solution to this system because there will never be a point that is simultaneously on both lines.

Teacher Tip: At this point, you could give them the definition of consistent and inconsistent equations.
Mathematical Practice: **Make sense of problems and persevere in solving them**. Mathematically proficient students will make conjectures and discuss why there is no solution.

Extension: Students can be asked to solve equations that have an infinite number of solutions, such as:

$x - 2y = 3$

$-2x + 4y = -6$

TI-Nspire Navigator Opportunity: *Quick Poll:*
See Note 6 at the end of this lesson.

Wrap Up

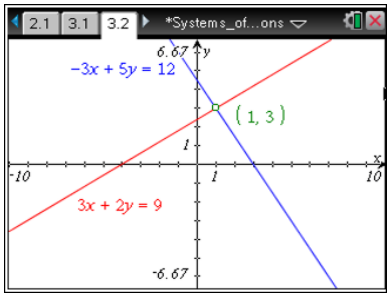
Upon completion of the discussion, the teacher should ensure that students understand:

- A solution to a system of equations consists of values of the variables that make all equations in the system simultaneously true.
- In the case of two equations and two variables, solutions consist of ordered pairs (or points) that simultaneously lie on the graphs of both equations.

Assessment

These problems could be used to assess student understanding:

1. The system of equations $\begin{cases} -3x+5y=12 \\ 3x+2y=9 \end{cases}$ is shown in the screen shot. From the screen shot solve the system and explain how you found your answer.



2. For the system of equations $\begin{cases} x+2y=1 \\ 7x-8y=-15 \end{cases}$, which of the following statements is true and how do you know?

   The point $(-1,1)$ is a solution to only one of the equations.
   The point $(-1,1)$ is a solution to the system of equations.
   The point $(-1,1)$ is a solution to none of the equations.

Solutions to Assessment:

1. The two lines intersect at the point $(1,3)$, so that is the solution to the system.
2. The point $(-1,1)$ is a solution to the system because it makes both equations true algebraically, That is, $-1+2=1$ and $-7-8=-15$.

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Note 1

Question 1 Class Capture: Take a Class Capture of page 1.2 where students have point *P* on different *x*-values. As a class, discuss the various cases that occur but how there are still similarities among the cases.

Note 2

Question 2 Quick Poll (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answer to question 2.

Note 3

Question 3 Class Capture: Take a Class Capture of page 1.2 where students have moved point P so that the two points on the lines have moved to the point of intersection (1, 2). As a class, discuss the various cases that occur.

Note 4

Question 3 Quick Poll (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answer to the algebraic portion of question 3.

Note 5

Questions 4-6 Quick Poll (Multiple Choice): Send a Multiple Choice Quick Poll, asking students to submit their answers to questions 4-6. The multiple choice questions could give them choice of the point satisfying one, both or neither of the equations.

Note 6

Questions 7-8 Quick Poll (*Open Response*): Send an Open Response Quick Poll, asking students to submit their answers to questions 7-8.