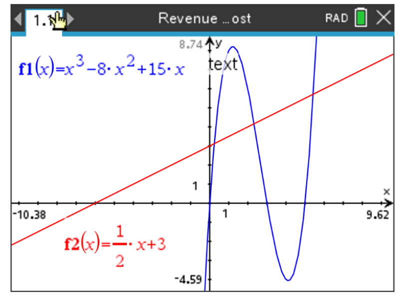
# Profit Equations and Calculus

**MATH NSPIRED**






## Math Objectives

- Students will further discuss the graphical and algebraic relationships between a function and its first and second derivatives, namely a profit function.
- Students will use their knowledge of Optimization to maximize profits or minimize losses.
- Students will use Integration to find a profit function given its rate of change.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

## Vocabulary

- Cost
- Profit
- Revenue
- Differentiation
- Optimization
- Integration

## About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 5 Calculus:
  **5.6b** (AI only) Solution of $f'(x) = 0$.
  **5.6c** (AI only) **5.8a** (AA only) Local maximum and minimum points
  **5.7** (AI only) **5.**8c (AA only) Optimization problems in context.
  **5.7a** (AA only) The second derivative
  **5.7b** (AA only) Graphical behaviour of functions, including the relationships between the graphs of $f, f', and\ f''$.
- As a result, students will:
  - Apply this information to real world situations

## TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

.

**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**
*Student Activity*
Profit_Equations_and_Calculus_Student-Nspire.pdf
Profit_Equations_and_Calculus_Student-Nspire.doc

## Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

To help with this activity, it should be known that Profit is equal to the Revenue of a manufactured item minus the item's Cost. In other words:

$$p(x) = r(x) - c(x)$$

Where $p(x)$ is the profit function, $r(x)$ is the revenue function, and $c(x)$ is the cost function.
Students should already know the relationship between a function and its first and second derivatives.

**Review**

(a) Discuss with a classmate how companies can use the first and second derivatives of the profit function to aid in the manufacturing and sales of their merchandise. Share your results with the class.

**Possible discussion points:** Finding the rate of change of the profit function, $p(x)$, or its first derivative, $p'(x)$, can aid in finding maximum profits and minimum losses. This can be achieved by finding solution(s) for $p'(x) = 0$. You can also use $p''(x)$ to verify which solutions are maximums or minimums.

(b) Discuss with a classmate the different techniques on the TI-Nspire CX II in which you can find and use the first and second derivatives (and their graphs) to aid in these profit function problems. Share your results with the class.

**Possible discussion points:** Using the Polynomial Roots Tool and the derivative function, graphing a derivative function and finding the zeros, using the Solve tool on the CAS version.

**Problem 1**

Armor Guard manufactures and sells silver polish. The cost of manufacturing the silver polish can be modelled by the function $c(x) = \frac{1}{2}x + 3$, where x is the number of cases, with each case containing 100 bottles, manufactured. Revenue is modelled by
$r(x) = x^3 - 8x^2 + 15x$. The company has enough work force to produce a max of a 1300 bottles per day.

(a) State the domain of both the cost and revenue functions.

**Solution:** For both functions, $0 \leq x \leq 13$, the company can produce a maximum of 1300 bottles, which is 13 hundred units.

(b) Without graphing, find the number of cases that the company should manufacture in order to maximize profits.

**Solution:** $p(x) = r(x) - c(x)$, and find the value where $p'(x) = 0$

$p(x) = (x^3 - 8x^2 + 15x) - \left(\frac{1}{2}x + 3\right)$

$= x^3 - 8x^2 + 14.5x - 3$

$p'(x) = 3x^2 - 16x + 14.5$

$0 = 3x^2 - 16x + 14.5$

Find the solution to this by using the poly root tool.

$x_1 = 4.18, x_2 = 1.16$

$p''(x) = 6x - 16$

$p''(4.18) > 0$ therefore $x_1$ is a minimum and $p''(1.16) < 0$ therefore $x_2$ is a maximum, since $x$ is in hundreds of units, the production level necessary to maximize profits is 116 bottles of silver polish.

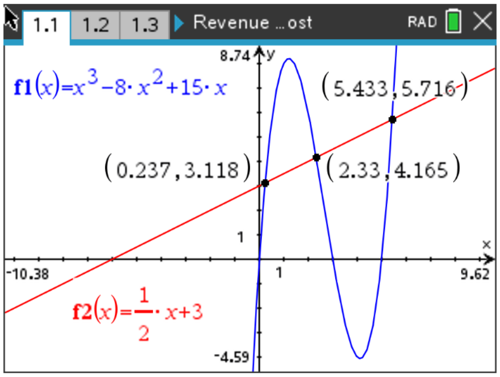
After years of developing, the company has found a way to now produce 250 cases per day.

(c) Find the number of cases that would cause the company to minimize profits (or maximize losses).

**Solution:** Since $p''(4.18) > 0$ therefore $x_1$ is a minimum, so at $x = 4.18$, the profit function $p$ has an absolute minimum in its domain. The production level that would maximize losses is 418 bottles of silver polish.

(d) Using a graphing utility, graph the cost and revenue functions, find the number of cases which should be manufactured if the company is going to just break even (i.e. the production level when $c(x) = r(x)$).

**Solution:** The company will break even at about 24 bottles, 233 bottles, and 543 bottles of silver polish.




(e) It would not maximize the company's profits to produce as many cases as workers are capable of producing. Use your graph to explain why.

**Solution:** The company's revenues decrease, despite the costs continuing to increase. This may be because of lack of demand for so many bottles of silver polish, which would mean they could not sell them all.

**Problem 2**

In its annual shareholders meeting Best Buds stated that the cost of producing earbuds is modelled by the function $c(x) = 3x + 1$ and the revenue for producing these earbuds is modelled by the function $r(x) = 40x + 12x^2 - x^3$ , where $x$ represents the number of batches of earbuds and each batch contains 100. Find the number of earbuds which maximizes profit and determine the maximum profit, in hundreds of dollars.

**Solution:** $p(x) = r(x) - c(x)$, and find the value where $p'(x) = 0$

$p(x) = (40x + 12x^2 - x^3) - (3x + 1)$

$= -x^3 + 12x^2 + 37x - 1$

$p'(x) = -3x^2 + 24x + 37$

$0 = -3x^2 + 24x + 37$

$x = 9.32, -1.32$

$p''(9.32) < 0$ therefore a maximum

$x = 9.32$ or 932 earbuds

Maximum Profit: 577 hundreds or $57,700

**Extension**

What if you were given the rate of change of a company's profit on the production of a certain item of merchandise, could you find the company's profit function? In this final problem of the activity, we will explore this situation.

**Possible discussion points:** We can find the profit function through integration.

**Problem 3**

Snow Shifters produces and sells shovels. The company's profit, $P(x)$ in thousands of dollars, changes based on the number of shovels produced per month.

The rate of change of their profit from producing shovels is modelled by $\frac{DP}{dx} = -4x + 15$, where $x$ is the number of shovels produces (in hundreds).

The company makes a profit of 18 (thousand dollars) when they produce 4 (hundred) shovels.

(a) Find an expression for $P$ in terms of $x$.

**Solution:**
$p(x) = \int(-4x + 15)\, dx = -2x^2 + 15x + c$
Since $y = 18\ when\ x = 4,$ solve for $c$.
$18 = -2(4)^2 + 15(4) + c$
$c = -10$
$p(x) = -2x^2 + 15x - 10$

(b) At certain times of year, the company has the ability to increase production. Describe how their profit changes if they increase production to over 5 (hundred) shovels and up to 6 (hundred) shovels.

**Solution:** Their profits decrease. Because the profit function is decreasing or the gradient is negative or the rate of change of $P$ is negative.

**Or**

$\int_5^6(-4x + 15)\, dx = -7$

**Or**

That finding $P(5) = 15$ and $P(6) = 8$.

## Further Discussion

As preparation for the end of course assessments, a portion of the exam is done with the calculator and a portion is done without. Using this activity as a guide, discuss with a classmate, and make a list of each method used throughout the activity and explain how to do them with and without a calculator. Remember, there are multiple ways to do each method with and without technology. Take time to discuss how to verify each process. Share your results with the class.

**Possible discussion points:**
There are many possible discussion points here. Finding a derivative by hand is a must, but finding solutions with and without a handheld is a great way to discuss and review for the year end assessments.

**TI-Nspire Navigator Opportunity:** ***Quick Poll (Open Response)***
**Given that $f(-7) = 5$, and $f'(x) = 4 - 6x$, find $f(x)$.**

**Teacher Tip:** Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review profit equations in calculus, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*