Graphs of Anti-derivatives

Student Activity

7 8 9 10 **11** **12**

TI-Nspire™ | Activity | Student | 50 min

Objective

Plot an antiderivative graph of a given function and make connections between the antiderivative graph and the original function graph.

Teacher Notes:

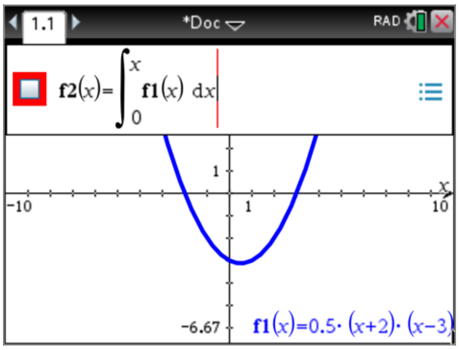
The TI-Nspire Teacher companion file accompanying this activity is titled: "Teacher Demonstration File". This file provides a point "P" that can be moved along the x-axis. As the point is moved the anti-derivate function is automatically generated for the curve: $f'(x)=0.5(x+1)(x-3)$. Move the point slowly so that text descriptions are displayed automatically to help prompt conversations. This file is intended for teachers to use at the beginning of the lesson to drive discussion and encourage students to make the connections between a function and its anti-derivative.

Exploration

Start a new TI-Nspire document and insert a Graph Application.

Enter the equation: $y=0.5(x+2)(x-3)$

By default this equation will be located in: $f_1(x)$. An anti-derivative of this function can be graphed using the definite integral. The definite integral template can be entered from the templates menu or by using the short cut combination: [Shift] + [+]



Note: The use of a 0 and x in the terminals will be explored later.

Question 1:

The anti-derivative graph for each of the following functions will be explored.

a. $y=0.5(x+2)(x-3)$ b. $y=x^3-2x^2+x-1$ c. $y=2\cos^2\left(\frac{x}{2}\right)$

d. $y=\frac{\sin(x)}{x}$ e. $y=200x\times 2^{-x}$ f. $y=|x|$

For each pair of graphs, comment upon and draw applicable region(s) for the original function and the graph of the anti-derivative where the original function:

- Crosses the *x*-axis from negative to positive
- Crosses the *x*-axis from positive to negative
- Has a turning point not touching the *x*-axis
- Has a turning point touching the *x*-axis
- Has a stationary point of inflection

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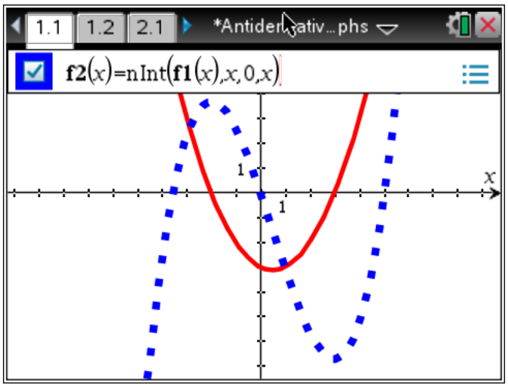
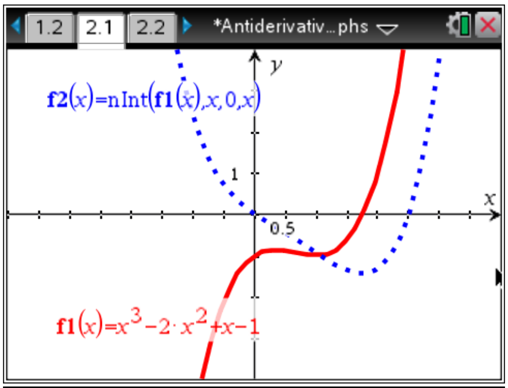
Derivative function: $y = 0.5(x+2)(x-3)$	**Primitive function:**
Crosses the x axis from negative to positive:	Function has a local minimum
Crosses the x axis from positive to negative:	Function has a local maximum
Turning point not touching the x axis:	Point of inflexion
Turning point touching the x axis.	Not Applicable
Has a stationary point of inflexion.	Not Applicable



Note:
Students can also use the Trace All option so that a vertical line aligns both the derivative and primitive functions. The line is moved using the left/right arrow keys on the navigation pad.

Derivative function: $y = x^3 - 2x^2 + x - 1$	**Primitive function:**
Crosses the x axis from negative to positive:	Function has a local minimum
Crosses the x axis from positive to negative:	Not Applicable
Turning point not touching the x axis:	Point of inflexion – There are two of these! (See notes)
Turning point touching the x axis.	Not Applicable
Has a stationary point of inflexion.	Not applicable



Note:
The local maximum and local minimum on the cubic function (derivative) generate two points of inflexion on the primitive function. These points may be hard for students to see or identify.
Zoom in using the zoom box option to draw focus to the two points in question and use a tangent line attached to the derivative function and move along the region:
$0 < x < 1.4$
For $x < \frac{1}{3}$ the tangent line is below the function.
At $x = \frac{1}{3}$ the tangent crosses the curve.
For $\frac{1}{3} < x < 1$ the tangent is above the curve. You must zoom in to see this precisely.
At $x = 1$ the tangent crosses the curve.
For $x > 1$ the tangent is below the curve again.

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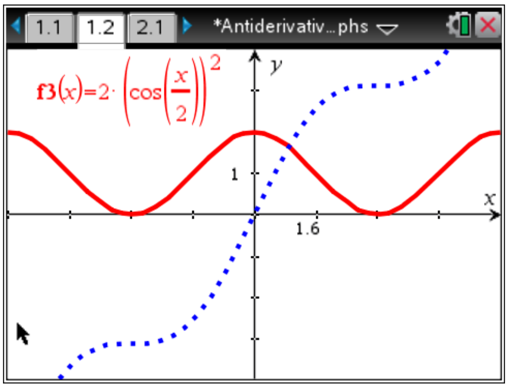
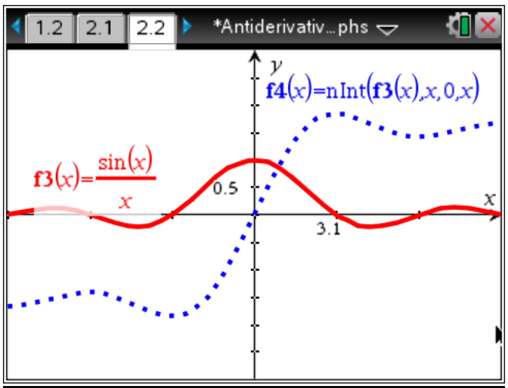
Derivative function: $y = 2\cos^2\left(\frac{x}{2}\right)$	**Primitive function:**
Crosses the x axis from negative to positive:	Not Applicable
Crosses the x axis from positive to negative:	Not Applicable
Turning point not touching the x axis:	Point of inflexion – infinitely many! (Equally spaced)
Turning point touching the x axis.	Stationary point of inflexion – infinitely many! (Eq.Sp)
Has a stationary point of inflexion.	Not applicable



Note:
The primitive function *resembles* an increasing function since the derivative can never be negative. Strictly speaking it is not correct to call it an increasing function since the derivative periodically equals zero. Nevertheless, the primitive function may be surprising to students as the derivative of simple trigonometric functions based on either sine or cosine produce similar trigonometric functions. Of course the function could be changed using an appropriate identity; however from a calculus perspective, focusing on the fact that the gradient function is always greater than or equal to zero is important.

Derivative function: $y = \frac{\sin(x)}{x}$	**Primitive function:**
Crosses the x axis from negative to positive:	Local minimum
Crosses the x axis from positive to negative:	Local maximum
Turning point not touching the x axis:	Point of inflexion – infinitely many! (Equally spaced)
Turning point touching the x axis.	Not applicable
Has a stationary point of inflexion.	Not applicable



Note:
As $x \to \pm\infty$ the gradient function approaches zero. This means the primitive function must approach a horizontal line in both directions.
For CAS enabled devices, students must use the numerical integral technique described in this investigation as there is no analytic solution to this integral.

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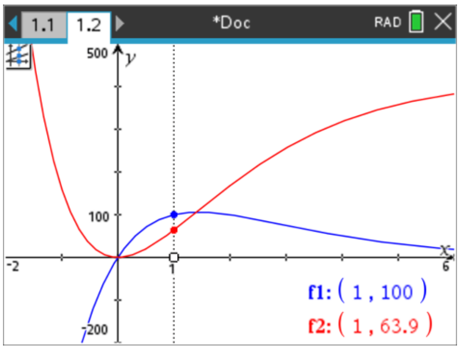
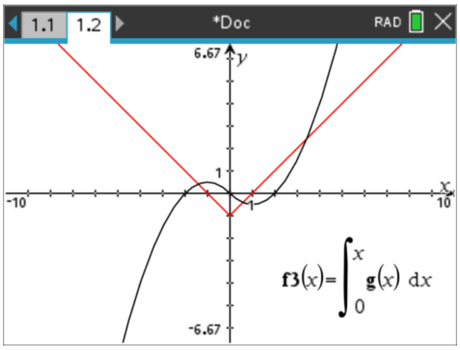
Derivative function: $y = 2^{-x} \cdot 200x$	**Primitive function:**
Crosses the x axis from negative to positive:	Local minimum
Crosses the x axis from positive to negative:	Not applicable
Turning point not touching the x axis:	Point of inflexion
Turning point touching the x axis.	Not applicable
Has a stationary point of inflexion.	Not applicable



Derivative function: $y = \lvert x \rvert - 1$	**Primitive function:**
Crosses the x axis from negative to positive:	Local minimum
Crosses the x axis from positive to negative:	Local maximum
Turning point not touching the x axis:	Point of inflexion
Turning point touching the x axis.	Not applicable
Has a stationary point of inflexion.	Not applicable



Note:
Students may believe that the primitive function is cubic, however the two separate branches are parts of a parabola, rotationally symmetrical about the origin.

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Calculator Tips

- When the equation in $f_1(x)$ is updated the anti-derivative updates automatically.
- Zoom Box or Zoom In / Out can be used to focus on specific areas of the graph.
- Graph labels can be automatically hidden via the Graph Application settings menu.

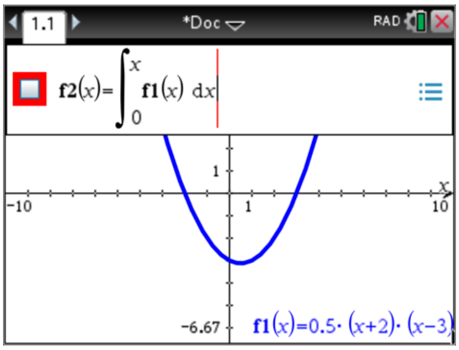
Extension

So far the purpose of the terminals has largely been ignored. Define the graph of $f_3(x)$ as:

$$\int_1^x f_1(x)dx$$

Define the graph of $f_4(x)$ as:

$$\int_{-1}^x f_1(x)dx$$



Question 2:

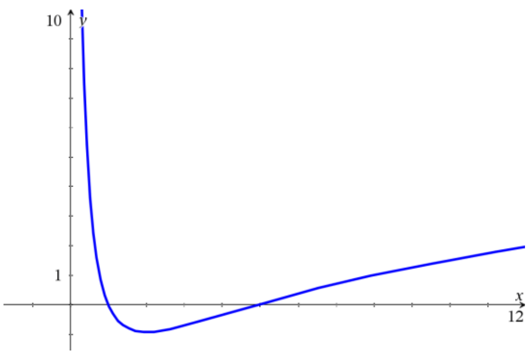
Comment on how the terminal(s) change the graph of the anti-derivative graph.

Changing the constant terminal has the effect of changing the constant of integration therefore applying a translation parallel to the y axis.

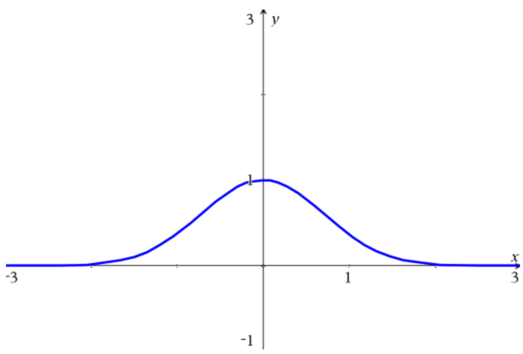
Sometimes we know the rate at which a function changes (derivative) but for a variety of regions we are unable to determine the corresponding anti-derivative. For the following two graphs draw the anti-derivative function, remember to cross-check your notes against the various applicable section of each curve.

Question 3:

a)



b)



Notes

Not knowing the equation for the function means students will have to use their prior learning in order to sketch the graph of the primitive function.

Notes

This 'bell shaped' curve closely resembles the normal distribution. The function has no analytical means of determining the integral. Students however should use the key points established in the first part of the investigation to determine an approximate function for the result.

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