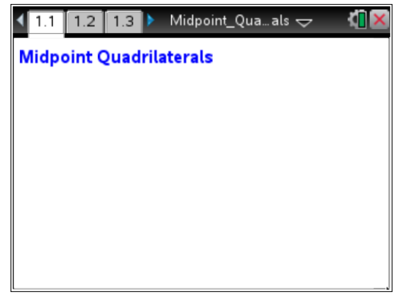
Math Objectives

- Students will explore the parallelogram formed by the midpoints of any quadrilateral.
- Students will further explore special outer and inner quadrilaterals formed by the connected midpoints. Area relationships will also be investigated.
- Students will prove geometric theorems about parallelograms.
- Students will use coordinates to prove simple geometric theorems algebraically.
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).
- Students will look for express regularity in repeated reasoning (CCSS Mathematical Practice).



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Vocabulary

- midpoint quadrilateral
- parallelogram
- convex quadrilateral
- concave quadrilateral
- midsegment

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- Make sure the users know how to attach a point to the grid on a graphs page.

About the Lesson

- The time varies for this activity depending on whether the TI-Nspire document is given or created by the students.
- Students can either create the TI-Nspire document by following the instructions given in *Midpoint_Quadrilaterals_Create.pdf* or they can use the pre-constructed document entitled *Midpoint_Quadrilaterals.tns*.
- This activity is intended to be introduced by the teacher and completed by the students as they discover and justify their conjectures.
- The student worksheet helps guide students through the activity and provides a place for students to record their answers.

Lesson Files:

Create Instructions

- Midpoint_Quadrilaterals_Create.pdf

Student Activity

- Midpoint_Quadrilaterals_Student.pdf
- Midpoint_Quadrilaterals_Student.doc

TI-Nspire document

- Midpoint_Quadrilaterals.tns

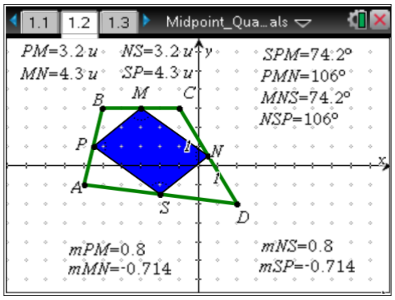
TI-Nspire™ Navigator™ System

- Class Capture
- Live Presenter

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand ( ). Press ctrl to grab the point and close the hand ( ).

Move to page 1.2.



Part 1 – Exploring the midpoint quadrilateral

On page 1.2, you will see an outer quadrilateral *ABCD* with an inner quadrilateral *PMNS* constructed using the midpoints of each side of the outer quadrilateral. Angle measurements, side measurements, and slopes of the sides of the inner quadrilateral are given.

Teacher Tip: As the activity progresses, you may want to review the following vocabulary: concave quadrilateral, convex quadrilateral, midpoint quadrilateral, midsegment.

The outer quadrilateral is attached to grid points. In case this was not done, use **Menu > Actions > Redefine.** First press on the point and then press on a grid point.

Move to any vertex of the outer quadrilateral and drag to explore. Observe the measurements as they change.

1. What type of quadrilateral is the inner quadrilateral? Explain how you know.

   Answer: The inner quadrilateral is a parallelogram because opposite sides are congruent, opposite angles are congruent, and slopes are equal so lines containing the sides are parallel.

2. Drag any of the outer vertices to form non-convex (concave) quadrilaterals. Is the inner quadrilateral still the same type as what you answered in problem 1?
   Answer: The inner quadrilateral remains a parallelogram no matter if the outer quadrilateral is concave or convex.

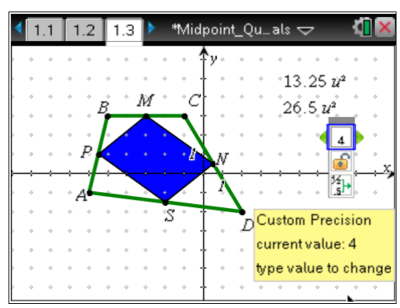
3. Is the inner quadrilateral ever a *special* quadrilateral (rectangle, square, and so on)? Drag the vertices to make the outer quadrilateral into the following shape types and record the type(s) of inner quadrilaterals that result. Make outer quadrilateral measurements if necessary.

Outer Quadrilateral	Inner Quadrilateral
Parallelogram	**Parallelogram**
Rectangle	**Rhombus**
Square	**Square**
Rhombus	**Rectangle**
Kite	**Rectangle**
Trapezoid	**Parallelogram**
Isosceles Trapezoid	**Rhombus**

Teacher Tip: If students choose to measure sides, make sure that the side is selected after pressing **Menu > Geometry > Measurement > Length**. If the side is not displayed, be sure to tab to the segment. Otherwise the perimeter will be measured.

The midpoint quadrilateral of any quadrilateral whose diagonals are equal is a rhombus. The midpoint quadrilateral of any quadrilateral whose diagonals are perpendicular is a rectangle. The midpoint quadrilateral of any quadrilateral whose diagonals are equal and perpendicular is a rhombus and a rectangle, and also a square. Encourage students to do necessary measurements to show special quadrilaterals. You might also want to divide these tasks between pairs of students.

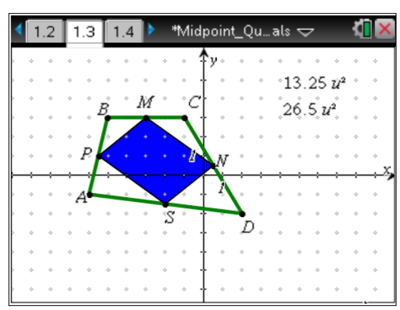
Move to page 1.3.



Part 2 – Comparing Areas

Press **Menu > Geometry > Measurement > Area**. Move to the inner polygon until the words *polygon MNSP* appear and press . Move the measurement to the upper right part of the screen and press . Repeat for outer quadrilateral *ABCD*. Then press esc to exit the **Measurement** tool.

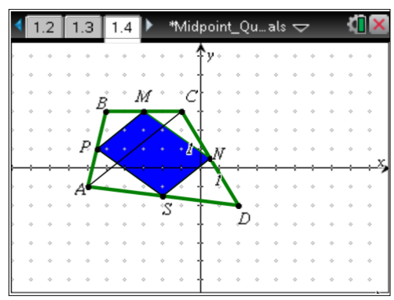
To increase the number of significant digits, hover the cursor above one of the area measures, press ctrl menu **> Attributes >** 4 **>** enter. Repeat for the other area measure.



4. What is the relationship between the areas of the inner and outer quadrilaterals?

Answer: Students should determine that the area of the outer quadrilateral is twice that of the inner quadrilateral.

> **Teacher Tip:** Beware of round off errors for this relationship. You may want to take students through the following calculation: The relationship can be easily observed by using the Text tool from the Actions menu. Use this to write the expression **a/b** on the page. Use the Calculate tool from the Actions menu by clicking first on the expression **a/b** and then move to each area measurement and click on each measurement. Press enter or to display the value.
>
> You may also want students to explore the areas of the smaller triangles with respect to other areas.



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Part 3 – Exploring Proof

Press **Menu > Geometry > Points & Lines > Segment**. Construct diagonal *AC* of the outer quadrilateral. Press esc to exit the **Segment** tool.

Press **Menu > Geometry Measurement > Length** to measure the length of the diagonal, *AC*, as well as the length of sides *SN* and *MP*. Press esc to exit the **Measurement** tool.

5. What is the relationship between the diagonal length and the inner quadrilateral side length?

Answer: The inner quadrilateral side length is $\frac{1}{2}$ the measurement of the diagonal.

Press **Menu > Geometry > Measurement > Slope** to measure the slope of the diagonal, *AC*, as well as the slope of sides *SN* and *MP*. Press esc to exit the **Measurement** tool. Drag the vertices of the outer quadrilateral.

6. What is the relationship between the slope of the diagonal and the slope of the midsegment?

Answer: The diagonal is parallel to both of the midsegments formed by a pair of opposite sides of the inner quadrilateral because the slopes are equal.

7. How could this be used to prove that the inner quadrilateral is a special type of quadrilateral?

Answer: The diagonal divides the original quadrilateral into two triangles. Two sides of the midpoint quadrilateral are midsegments of the triangles. This means they are both parallel to the diagonal and half as long. If one pair of opposite sides of a quadrilateral is equal in length and parallel, the quadrilateral is a parallelogram. The other diagonal can be drawn to show that the other pair of sides is also parallel and congruent.

Extension – Exploring More Midpoint Quadrilaterals

Have students construct more midpoint quadrilaterals and look for patterns.
Are there any special properties for the midpoint polygons (formed by connecting consecutive midpoints) for other types of polygons?

Sample answer: Successive midpoint quadrilaterals are similar; that is the third midpoint quadrilateral is a parallelogram similar to the first, the fourth is similar to the second, and so on. These parallelograms converge on the point of intersection of segments connecting midpoints of opposite sides.