Reflection Properties of Conics

Math Objectives

- Students will be able to state the reflection property for a flat surface in terms of the equality of angle of incidence and angle of reflection and apply it to a curved surface (using the tangent line as a flat surface).
- Students will be able to describe the special distance relationships satisfied by any point on each of the three conic sections (parabola, ellipse, and hyperbola) in terms of their foci and/or directrix.
- Students will be able to describe the reflective (carom) properties of each of the three conic sections (parabola, ellipse, and hyperbola) in terms of their foci and/or directrix.
- Students will model with mathematics (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

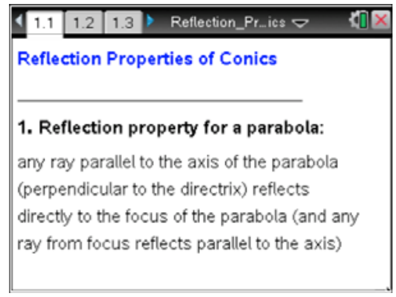
- conic sections (parabola, ellipse, hyperbola)
- tangent line
- angle of incidence
- angle of reflection

About the Lesson

- This lesson involves investigating the properties of basic reflective principles of conics.
- As a result, students will:
  - Manipulate the source of the ray and its point of contact with the conic to observe its effect on the reflected ray.
  - See the special reflective properties of each conic in terms of foci and/or directrix.
  - Consider applications of the reflective properties.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Reflection_Properties_of_Conics_Student.pdf

Reflection_Properties_of_Conics_Student.doc

TI-Nspire document

Reflection_Properties_of_Conics.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

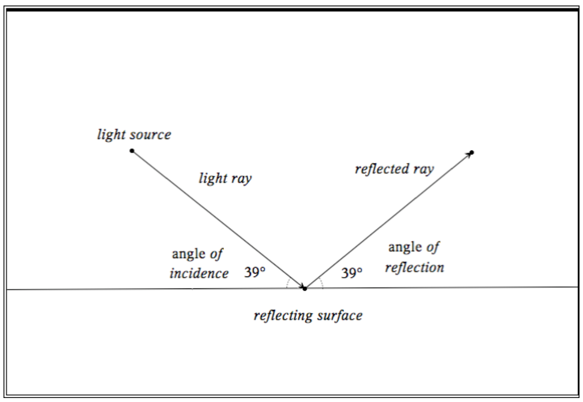
1. *Reflections off a flat surface.*

When light reflects off a flat mirror or a ball caroms off a flat wall, the *angle of incidence* is equal to the *angle of reflection*.

- The angle the ray of light makes with the surface of the mirror or the angle the straight path of the ball makes with the wall is the *angle of incidence*.
- The angle the ray of reflected light or the angle of the carom path of the ball makes with the wall is the *angle of reflection*.

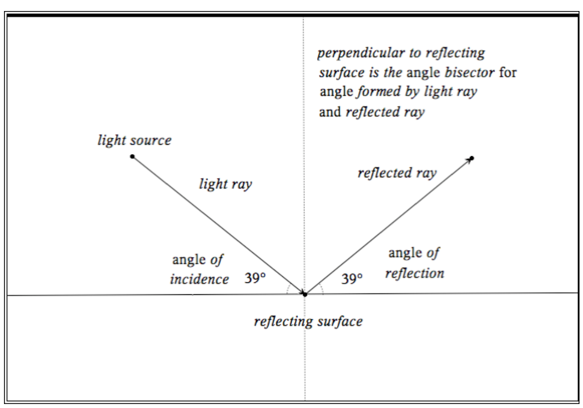
a. Draw the path of the reflected ray in the picture below.

Sample Answer:



b. If a perpendicular were drawn to the reflecting surface at the point of contact, what would its relationship be to the angle formed by the light ray and its reflected ray?

Sample Answer:



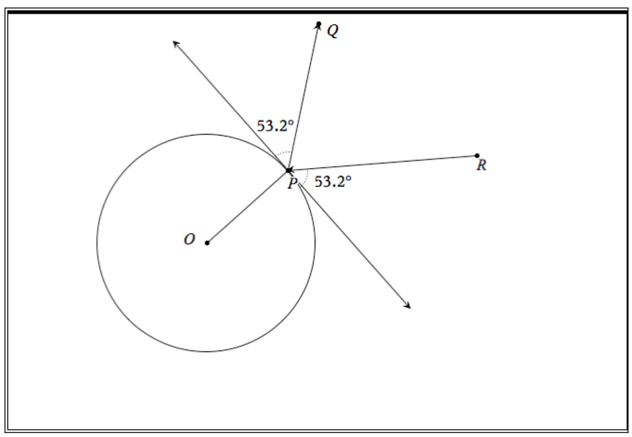
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

2. *Reflections off a curved surface.*

What happens when a light ray reflects off a curved mirror or a ball caroms off a curved wall? The reflection property is the same where the tangent line to the curve at the point of impact is the reflecting surface. For example, if you bounced a ball off a circular wall, it would bounce in the same direction as if it bounced off a straight wall along the tangent line at that same point.

In the picture below, a circular wall with center *O* is shown along the path of a ball striking the wall at point *P* from its starting point at *R.* Sketch the carom path of the ball, and describe how you could determine it precisely.

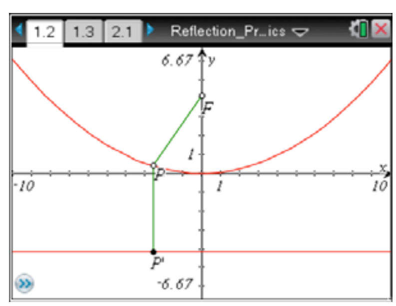
Sample Answer: Since the tangent to a circle at a point is perpendicular to the radius drawn to the same point, one could construct a tangent line first and then measure the angle of incidence that segment *RP* makes with that tangent. The reflecting ray *PQ* must make the same angle of reflection (see below).



- For more general curved surfaces, this reflection property works in the same way – *the angle of incidence to the tangent line is the equal to the angle of reflection to the tangent line.*

Reflective surfaces shaped like conics have remarkable properties.

Move to page 1.2.



3. This page shows the graph of a parabola with focus *F* on the positive y-axis and horizontal directrix. The point *P* can be grabbed and moved along the parabola. The point *P'* is located on the directrix so that segment *PP'* is perpendicular to the directrix. (If you move the focus *F,* you will also change the

directrix and the shape of the parabola.)

a. Measure segments *FP* and *PP'*. Drag the point *P*. What is special about the relationship between these two segments for any parabola?

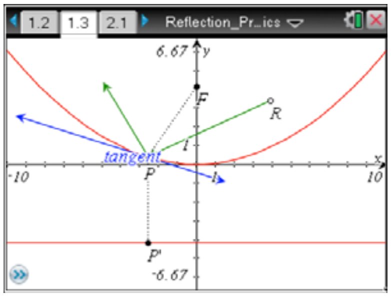
Sample Answers: The two segments *FP* and *PP'* always have the same length.

Tech Tip: To measure the length of a line segment, select **Measurement > Length**, and point and click on the segment to record its length. (Use esc to exit this tool.).

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: Remind students that a defining property for a parabola is the set of points *P* satisfying *FP* = *PP'*.

Move to page 1.3.

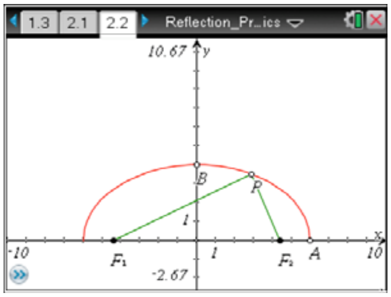


b. Here is the same parabola with a tangent line at point *P* shown. A ray of light from point *R* reflects off the parabola. Move point *R* around to see the different reflected rays. What is special when *R* = *F*?

Sample Answers: When *R* = *F* the reflected ray is always perpendicular to the directrix and parallel to the axis of the parabola.

Note: Car headlights have mirrored surfaces in the shape of paraboloids (every cross-section through the center is the same parabola). For "high beams" a bulb located at the common focus of all the parabolas is lit. All of the reflected light rays from the mirrored surface point straight out!

Move to page 2.2.



4. This page shows the graph of the top half of an ellipse with foci F_1 and F_2 on the x-axis. The point *P* can be grabbed and dragged along the ellipse. (You can change the shape of the ellipse by moving points *A* or *B* to change the length of the major or minor semi-axis length.)

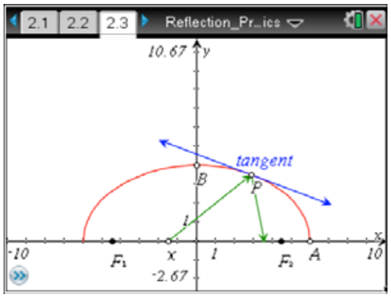
a. Measure segments F_1P and F_2P and add their lengths. Drag the point *P*. What is special about the relationship between the lengths of these two segments for any ellipse?

Sample Answers: The lengths of the two segments F_1P and F_2P always have the same sum.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: Remind students that this a defining property for an ellipse is the set of points *P* for which $F_1P + F_2P$ is constant.

Move to page 2.3.

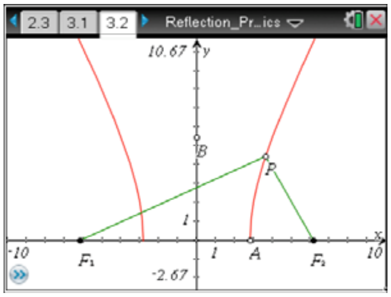


b. Here is the same ellipse with a tangent line at point *P* shown. A ray of light from point *x* reflects off the ellipse. Move point *x* around to see the different reflected rays. What is special when $x = F_1$ or F_2?

Sample Answers: When $x = F_1$ or F_2, the reflected ray always points directly at the other focus.

Note: A "whispering room" has walls and ceilings in the shape of an ellipse rotated about a common axis (with two common foci F_1 and F_2). If one person stands at one focus and whispers, a person standing far away at the other focus can easily hear it while those close to the speaker might not. The reason is that all of the sound waves emanating from the speaker bounce off the walls toward a single common point – the other focus! There are also special billiard tables in the shape of an ellipse that have a single pocket located at one focus. If one shoots a ball from the other focus in any direction, it will carom off the railing directly into the pocket!

Move to page 3.2.



5. This page shows the graph of the top half of two branches of a hyperbola with foci F_1 and F_2 on the x-axis. The point *P* can be grabbed and dragged along one branch of the hyperbola. (You can change the shape of the hyperbola by moving points *A* or *B*.)

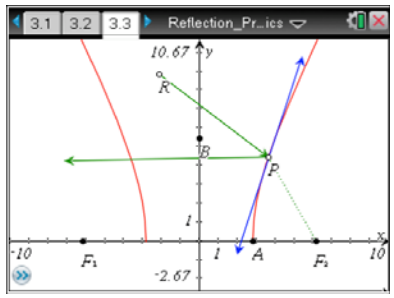
a. Measure segments F_1P and F_2P and calculate the difference of their lengths. Drag the point *P*. What is special about the relationship between the lengths of these two segments for any hyperbola?

Sample Answers: The difference of the two segment lengths F_1P and F_2P is always the same.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Teacher Tip: Remind students that a defining property for a hyperbola is the set of points *P* for which $|F_1P - F_2P|$ is constant.

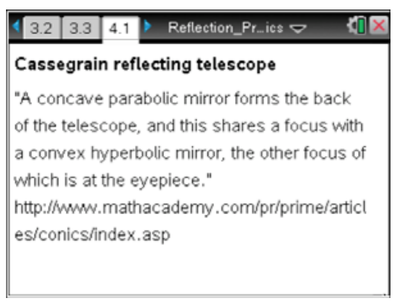
Move to page 3.3.



b. Here is the same hyperbola with a tangent line at point *P* shown. A ray of light from point *R* reflects off the hyperbola at point *P*. Move point *R* around to see the different reflected rays. What is special when the ray *RP* points directly at F_2?

Sample Answers: When the ray *RP* points directly at F_2 the reflected ray points directly at F_1

Teacher Tip: Some students might move point *R* to the same side of the hyperbola as the focus. Remind them that we are looking for ray *RP* to point *at* the focus F_2 (i.e., *P* will be *between* point *R* and F_2).

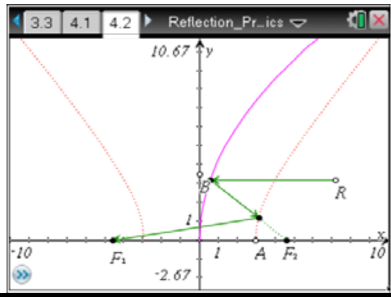


Note: The reflective properties of a parabola and a hyperbola are combined in spectacular fashion to make reflecting telescopes. **Read how on page 4.1.**

When the telescope is pointed at a distant object, all of the light rays from that object are practically parallel to the axis of the parabolic mirror (because of the great distance). Hence, these light rays all bounce off the sides of the telescope to the focus of the parabola.

However, before the light rays can hit that focus, they reflect off a hyperbolic mirror having one focus the same as the parabola and the other focus at the eye of the spectator.

Move to page 4.2.



See a dynamic illustration of the telescope on this page. Move the source point *R* anywhere in the "bowl" of the telescope and note how the combined reflections always go directly to the eyepiece.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The basic reflective principle (angle of incidence = angle of reflection) for a flat surface and how it extends to a curved surface having a tangent line.
- The basic defining distance relationships for each of the three conics.
- The special reflective properties of each of the three conic sections.

Assessment

Give students a reflective surface described as a conic section and the description of a light ray from a focal source and ask where the reflecting ray will point.

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Note 1

Name of Feature: Quick Poll
A *Quick Poll* can be used to gather student responses to 1b.

Note 2

Name of Feature: Quick Poll
A *Quick Poll* can be used to gather student responses to the special defining relationship for a parabola, ellipse, and hyperbola.

Note 3

Name of Feature: Screen Capture
Alternatively, for each conic reflection page of the .tns file (1.3, 2.3, 3.3), the teacher could ask the students to place the light source in a position that makes the reflecting ray point in a particular direction (example: toward the other focus) and take a screen capture to highlight the special property.