Math Objectives

Students will be able to identify the correct 2 × 2 matrix that, when multiplied to a matrix representation of a polygon, results in a polygon:

- Reflected across the *x*-axis.
- Reflected across the *y*-axis.
- Rotated 90° about the origin.
- Rotated 180° about the origin.

Students will look for regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- rotation
- reflection
- matrix
- element
- identity

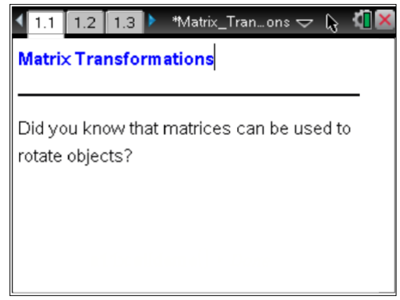
About the Lesson

This lesson involves:

- Grabbing vertices of a polygon undergoing reflections and rotations in the coordinate plane to determine the transformation's type.
- Duplicating a reflection or rotation by:
  - Changing the elements of 2 × 2 matrices.
  - Evaluating the effects of the multiplying matrix on the polygon in the coordinate plane.
- As a result, students will make conjectures about the relationships between certain 2 × 2 matrices and their effects on the resulting transformed polygons obtained through matrix multiplication.

TI-Nspire™ Navigator™ System

- Use Live Presenter for student demonstrations.
- Use Screen Capture to examine patterns that emerge.
- Use Quick Polls to check for student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a slider
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- In Graphs, you can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Matrix_Transformations_Student .pdf

Matrix_Transformations_Student .doc

TI-Nspire document

Matrix_Transformations.tns

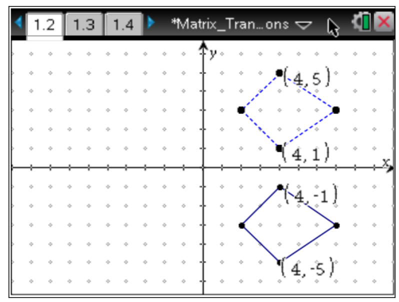
Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: ***Live Presenter, Quick Poll (Open Response)***
See Note 1 at the end of this lesson.

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text.* Then press ctrl [ ] to grab the point and close the hand ( ).

Move to page 1.2.



1. Grab and move a vertex of the polygon in Quadrant I.
   a. How are the polygons in Quadrants I and IV related?

      <u>Answer:</u> They are reflections over the *x*-axis of each other.

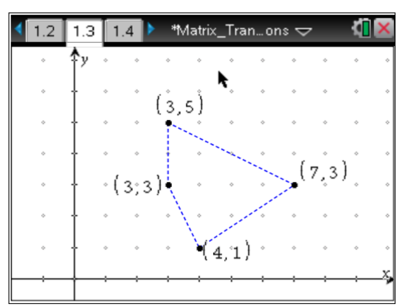
   b. If the coordinates of a vertex in Quadrant I are (3, 9), what are the coordinates of the corresponding vertex in Quadrant IV?

      <u>Answer:</u> (3,–9)

   c. If the coordinates of a vertex in Quadrant I are (*x*, *y*), what are the coordinates of the corresponding vertex in Quadrant IV?

      <u>Answer:</u> (*x*, –*y*).

Move to page 1.3.



2. Every polygon has a matrix representation of $\begin{bmatrix} x_1 & x_2 & x_3 & \dots \\ y_1 & y_2 & y_3 & \dots \end{bmatrix}$

   Write the matrix representation of the polygon in Quadrant I.

   <u>Answer:</u> $\begin{bmatrix} 3 & 3 & 4 & 7 \\ 5 & 3 & 1 & 3 \end{bmatrix}$

Teacher Tip: The next question, and others that follow, require knowledge of matrix multiplication. To trigger previous learning, you may want to display the following matrix multiplication.

$$\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix}=\begin{bmatrix} 2(1)+1(2) & 2(2)+1(1) & 2(3)+1(5) \\ 3(1)+0(2) & 3(2)+0(1) & 3(3)+0(5) \end{bmatrix}=\begin{bmatrix} 4 & 5 & 11 \\ 3 & 6 & 9 \end{bmatrix}$$

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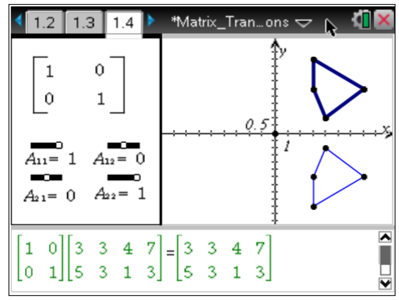


3. The polygon in Quadrant I has its matrix notation displayed at the bottom of the screen and is being multiplied by the displayed matrix. The product of the matrices is also displayed.

   a. Why does multiplying by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ result in an identity?

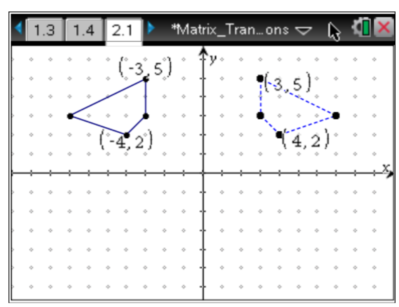
   Answer: Each newly calculated *x*-value is determined by multiplying each *x*-coordinate by 1, the corresponding *y*-coordinate by zero, and adding these together. Therefore, 1(*x*) + 0(*y*) = *x*. Similarly, each newly calculated *y*-value is determined by multiplying each *x*-coordinate by 0, the corresponding *y*-coordinate by 1, and adding these together. Therefore, 0(*x*) + 1(*y*) = *y*.

   b. Grab and move the sliders for each element of the 2 × 2 matrix until the polygon in Quadrant I is a reflection of the polygon in Quadrant IV. What 2 × 2 matrix results in a reflection over the *x*-axis?

   Answer: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 2 at the end of this lesson.

Move to page 2.1.



4. Grab and move a vertex of the polygon in Quadrant I.

   a. How are the polygons in Quadrants I and II related?

   Answer: They are reflections over the *y*-axis of each other.

b. If the coordinates of a vertex in Quadrant I are (12, 1), what are the coordinates of the corresponding vertex in Quadrant II?

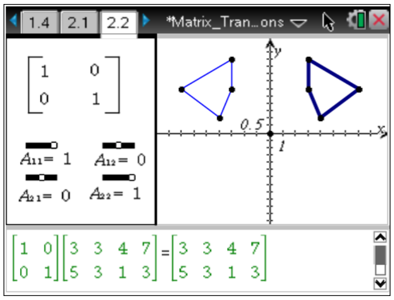
Answer: (–12, 1)

c. If the coordinates of a vertex in Quadrant I are (*x*, *y*), what are the coordinates of the corresponding vertex in Quadrant II?

Answer: (–*x*, *y*).

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response), Live Presenter*
See Note 3 at the end of this lesson.

Move to page 2.2.



5. Grab and move the sliders for each element of the multiplication matrix until the polygon in Quadrant I is a reflection of the polygon in Quadrant II.

a. What 2 × 2 matrix results in a reflection over the *y*-axis?

Answer: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

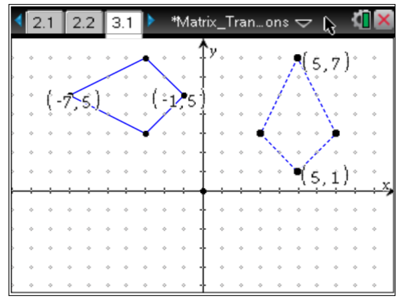
b. Why does this matrix multiplication result in a reflection over the *y*-axis?

Answer: Each newly calculated *x*-value is determined by multiplying each *x*-coordinate by –1, the corresponding *y*-coordinate by zero, and adding these together. Therefore, $-1(x)+0(y)=-x$.

Similarly, each newly calculated *y*-value is determined by multiplying each *x*-coordinate by 0, the corresponding *y*-coordinate by 1, and adding these together. Therefore, $0(x)+1(y)=y$.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.

Move to page 3.1.



6. Grab and move a vertex of the polygon in Quadrant I.
   a. How are the polygons in Quadrants I and II related?

      Answer: The polygon in Quadrant II is a 90° counterclockwise rotation of the polygon in Quadrant I.

   b. If the coordinates of a vertex in Quadrant I are (3, 9), what are the coordinates of the corresponding vertex in Quadrant II?

      Answer: (–9, 3)

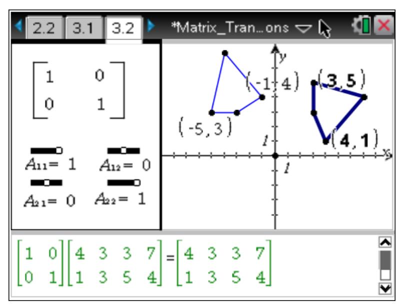
   c. If the coordinates of a vertex in Quadrant I are (*x*, *y*), what are the coordinates of the corresponding vertex in Quadrant II?

      Answer: (–*y*, *x*).

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response), Live Presenter*
See Note 5 at the end of this lesson.

Teacher Tip: By definition, rotations in mathematics are counterclockwise unless otherwise stated.

Move to page 3.2.



7. Grab and move the sliders for each element of the multiplication matrix until the polygon in Quadrant I is a rotation of the polygon in Quadrant II.

   a. What 2 × 2 matrix results in a 90° rotation about the origin?

      Answer: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

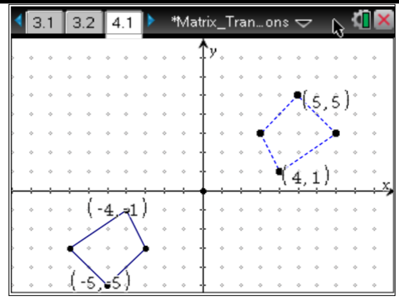
   b. Why does this matrix multiplication result in a 90° rotation about the origin?

      Answer: Each newly calculated *x*-value is determined by multiplying each *x*-coordinate by 0, the corresponding *y*-coordinate by –1, and adding these together. Therefore, $0(x) - 1(y) = -y$.

      Similarly, each newly calculated *y*-value is determined by multiplying each *x*-coordinate by 1, the corresponding *y*-coordinate by 0, and adding these together. Therefore, $1(x) + 0(y) = x$.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 6 at the end of this lesson.

Move to page 4.1.



8. Grab and move a vertex of the polygon in Quadrant I.
   a. How are the polygons in Quadrants I and III related?

      Answer: The polygon in quadrant II is a 180° counterclockwise rotation of the polygon in Quadrant I.

   b. If the coordinates of a vertex in Quadrant I was (3, 9), what are the coordinates of the corresponding vertex in Quadrant III?

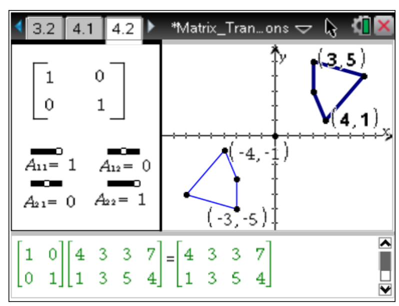
      Answer: (–9,–3)

   c. If the coordinates of a vertex in Quadrant I was (x, y), what are the coordinates of the corresponding vertex in Quadrant III?

      Answer: ($-x$, $-y$).

TI-Nspire Navigator Opportunity: ***Quick Polls (Open Response), Live Presenter***
See Note 7 at the end of this lesson.

Move to page 4.2.



9. Grab and move the sliders for each element of the multiplication matrix until the polygon in Quadrant I is a rotation of the polygon in Quadrant III.

   a. What 2 × 2 matrix results in a 180° rotation about the origin?

      Answer: $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b. Why does this matrix multiplication result in a 180° rotation about the origin?

Answer: Each newly calculated *x*-value is determined by multiplying each *x*-coordinate by –1, the corresponding *y*-coordinate by 0, and adding these together. Therefore, $-1(x)+0(y)=-x$.

Similarly, each newly calculated *y*-value is determined by multiplying each *x*-coordinate by 0, the corresponding *y*-coordinate by –1, and adding these together. Therefore, $0(x)-1(y)=-y$.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 8 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure students know that when a polygon with vertices (x_1,y_1), (x_2, y_2), … is written as a matrix and multiplied by a 2 × 2 matrix, the result is a:

- Reflection over the *x*-axis when the 2 × 2 matrix is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.
- Reflection over the *y*-axis when the 2 × 2 matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
- Rotation of 90° centered at the origin when the 2 × 2 matrix is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
- Rotation of 180° centered at the origin when the 2 × 2 matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

Assessment

1. Multiplying the matrix representation of a polygon by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ results in what transformation?

   Answer: A reflection across the *x*-axis.

2. Multiplying the matrix representation of a polygon by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ results in what transformation?

   Answer: A rotation of 180°.

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Note 1

Question 1a, *Live Presenter:* Consider using *Live Presenter* to demonstrate how to grab and move a point of the polygon in Quadrant 1.

Question 1b, *Quick Poll (Open Response), Live Presenter:* Consider using a *Quick Poll* for students to submit their answer to check for understanding. For students having trouble, consider using *Live Presenter* to demonstrate the concept.

Note 2

Question 3b, *Screen Capture:* Use a *Screen Capture* of page 1.3. As a class, discuss the relationship between the elements of the 2×2 matrix, the resulting matrix, and its graph. If students have difficulty, remind them that they are trying to make the *x*-coordinates remain the same, but the *y*-coordinates have the opposite sign.

Note 3

Question 4b, *Quick Poll (Open Response), Live Presenter:* Consider using a *Quick Poll* for students to submit their answer to check for understanding. For students having trouble, consider using *Live Presenter* to demonstrate the concept.

Note 4

Question 5b, *Screen Capture:* Use a *Screen Capture* of page 2.2. As a class, discuss the relationship between the elements of the 2×2 matrix, the resulting matrix, and its graph. If students have difficulty, remind them that they are trying to make the *y*-coordinates remain the same, but the *x*-coordinates have the opposite sign.

Note 5

Question 6b, *Quick Poll (Open Response), Live Presenter:* Consider using a *Quick Poll* for students to submit their answer to check for understanding. For students having trouble, consider using *Live Presenter* to demonstrate the concept.

Note 6

Question 7b, *Screen Capture:* Use a *Screen Capture* of page 3.2. As a class, discuss the relationship between the elements of the 2×2 matrix, the resulting matrix, and its graph. If students have difficulty, remind them of their answer to question 6b.

Note 7

Question 8b, *Quick Poll (Open Response), Live Presenter:* Consider using a *Quick Poll* for students to submit their answers to check for understanding. For students having trouble, consider using *Live Presenter* to demonstrate the concept.

Note 8

Question 9b, *Screen Capture:* Use a *Screen Capture* of page 4.2. As a class, discuss the relationship between the elements of the 2×2 matrix, the resulting matrix, and its graph. If students have difficulty, remind them of their answer to question 8b.