

World Population

ID: 9321

 Time required
 45 minutes

Activity Overview

In this activity, students use their graphing calculator to explore world population data from the years 1950-2006. They will develop various equations to model the data.

Topic: Exponential Functions

- Evaluate the exponential function $f(x) = Aa^x$ for any value of x .
- Calculate the exponential line of best fit to model bivariate data and use it to predict a value of one variable corresponding to a value of the other.

Teacher Preparation and Notes

- This activity is designed for use in a Pre-Calculus classroom.
- Prior to beginning the activity, students should download the **WORLDPOP** program to their graphing calculators.
- Students should begin by clearing out any functions from the $\boxed{Y=}$ screen and turning off all Stat Plots.
- They should also have some experience with the general form of an exponential function $f(x) = a \cdot b^x$, be familiar with base e , know the laws of exponents, and experience functions in the form of $f(x) = a \cdot e^x$.
- Before the activity you may choose to share the following information to generate interest:

The world population is well over the 6 billion mark with China at 1,319,175,347 people and India at 1,118,088,730 people compared to the US of 303,665,014.

The population density of China 137 people/km² and India is 336 people/km² compared to the US of 31 people/km².

*Source: U.S. Census Bureau, Population Division/International Programs Center
www.census.gov/popest/estimates*

- **To download the calculator program WORLDPOP and student worksheet, go to education.ti.com/exchange and enter "9321" in the keyword search box.**

Associated Materials

- *WorldPopulation_Student.doc*
- *WORLDPOP.8xp (program)*

Population growth can be modeled using various exponential functions. You are going to model the data by using two different methods to find the best model. The data that you are going to investigate is the midyear world population from the years 1950-2006. The years have been modified so that year 1 represents year 1950.

Source: U.S. Census Bureau, Population Division/International Programs Center

To start, students will need an empty graph window. Clear out any functions from the $\boxed{Y=}$ screen and turn off all Stat Plots.

Problem 1 – Find an exponential equation by hand using two points.

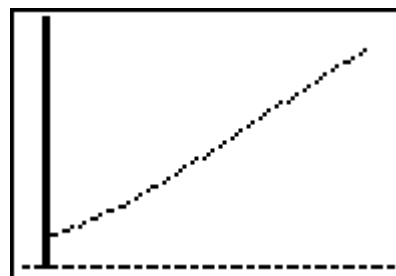
First students will use two points to find an equation to model the data. After running the **WORLDPOP** program, the data will be listed in L1 (the year) and L2 (the population).

L1	L2	L3	1
1	2.56E9	-----	
2	2.59E9		
3	2.64E9		
4	2.68E9		
5	2.73E9		
6	2.78E9		
7	2.83E9		

L1(1) = 1

Make sure that students understand that 1 = 1950, 2 = 1951, etc.

By observing the data in the scatter plot, students should determine that an exponential function, $y = a(b^x)$ where a is the initial value and b is the multiplier or in this case the growth rate, would be the best model. Note: Some students may think that the data is linear. Discuss with the class why this is not so.



Students are to pick two points using $\boxed{\text{TRACE}}$.

They will then set up two equations using the points and divide them to find the value of b .

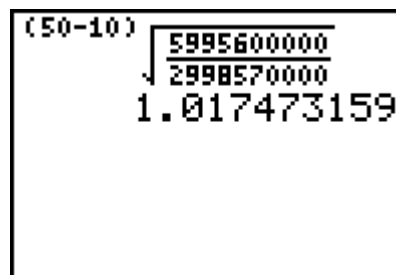
$$\frac{y_2}{y_1} = \frac{a(b^{x_2})}{a(b^{x_1})} \rightarrow b = \sqrt[x_2 - x_1]{\frac{y_2}{y_1}}$$

Note: Students should enter the value of the root before selecting $\sqrt[x]{}$.

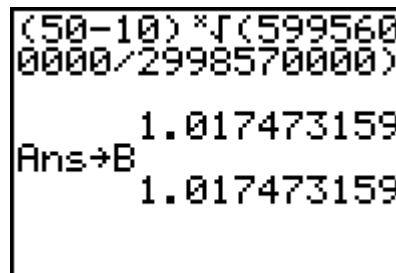
If using Mathprint OS:

When students select $\sqrt[x]{}$ from $\boxed{\text{MATH}}$, the cursor will go under the radical sign. After entering the radicand, press $\boxed{\rightarrow}$ to move out from under the radical sign.

Also to enter $\frac{y_2}{y_1}$ as a fraction instead of using the division key, press $\boxed{\text{ALPHA}} \boxed{[F1]}$ and select **n/d**. Enter the numerator, press $\boxed{\downarrow}$, and then enter the denominator.

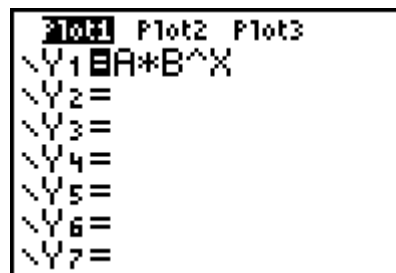


Store this value by pressing $\boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{B}} \boxed{\text{ENTER}}$. Students' answers will vary depending on the points they choose, but their answer should be just above 1. This means that the population is growing about 1.02%.



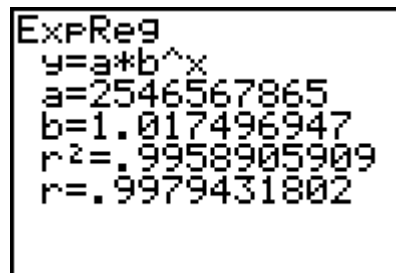
Then students will substitute the value of b back into one of the equations and solve for a . Once again, students' answers for initial population will vary depending on the points they choose.

Students will now graph their exponential equation with the data and use it to find the population in 2015 and in 1890. They can do this by pressing $\boxed{\text{TRACE}}$ and enter the values. The y -value is the population.



Problem 2 – Finding the exponential regression.

Now students find an exponential regression equation, using the **ExpReg**, command to fit the data. This will output the values of a and b and store the equation in Y_2 . The values of r^2 and r are not relevant for this activity.

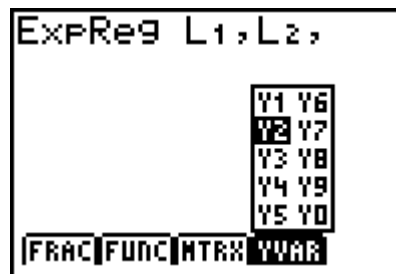


Note: Warn students to take care not to select Y_1 since their equation found in the first part is stored there.

Students should all have the same regression equation. They are to answer the questions on the worksheet using this equation.

If using Mathprint OS:

Students can enter Y_2 using the shortcut menu. To do this, press $\boxed{\text{ALPHA}} \boxed{\text{F4}}$ and select Y_2 .



Solutions
Part 1

All answers will vary. Sample answers:

- The data looks somewhat linear or exponential.
- Two points: (10, 2.99857E9) and (50, 5.9956E9)
- Two sample equations:
 $2998560000 = a(b^{10})$ and $5995600000 = a(b^{50})$
- $\frac{5995600000}{2998570000} = \frac{a(b^{50})}{a(b^{10})}$
- The a values divide out.
- $b = 1.017473159$
- The percent growth is 1.017473159%
- $a = 2,521,648,662$
- $y = 2,521,648,662(1.017473188^x)$
- The model fits well in the beginning but now as well at the end of the data.
- In year 2015, population is 7,774,582,441 and in year 1890 it was 891,880,950.

Part 2

- $y = 2546567865(1.017496947^x)$
- 2,546,567,865
- 1.017496947%
- The exponential regression has the initial population at about 25 million more people and the percentage of growth at .0023759 more.
- The exponential regression again fits very well in the beginning but goes up even sharper at the end.
- In year 2015, population is 7,863,352,008 and in year 1890 it is 899,432,063.
- For interpreting the population in the earlier years, I would use the exponential regression; however, to predict the population in the later years, I would use my own equation.
- To find the year when the population reaches 8.5 billion, I would change my window settings and graph a line at $y = 8,500,000,000$ and then find the intersection point between the two graphs by pressing $\boxed{2nd}$ [CALC] and selecting **5:Intersect**.
- The year is approximately 2019.488.