Roller Coaster Ride

Math Objectives

Students will be able to:

- Relate the first derivative of a function to its critical points and identify which of these critical points are local extrema.
- Visualize why the first derivative test works and how it is used to determine local minima and maxima.

Vocabulary

- first derivative
- critical point
- local maximum, minimum, extrema

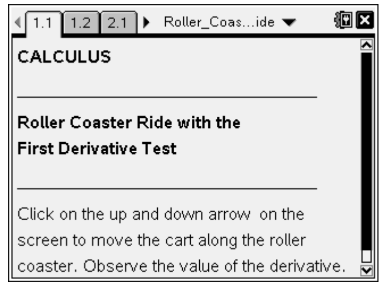
About the Lesson

- This lesson is a follow-up lesson to the *First Derivative Test* activity.
- This lesson involves using the first derivative test in a real-world application.

Related Lessons

Prior to this lesson: First Derivative Test

After this lesson: What Is a Slope Field?



TI-Nspire™ Technology Skills:

- Download TI-Nspire document
- Open a document
- Move between pages
- Move a slider bar
- Grab and drag points along a graph

Tech Tips:

- *Make sure the **font size** on your TI-Nspire handhelds is set to Medium.*

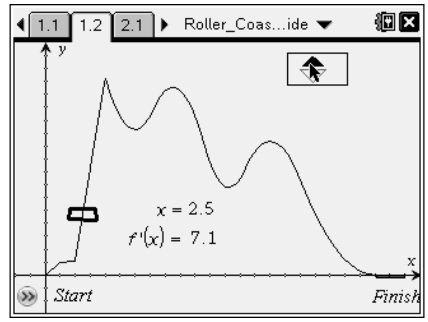
Lesson Materials:

Student Activity
Roller_Coaster_Ride_Student.PDF
Roller_Coaster_Ride_Student.DOC

TI-Nspire document
Roller_Coaster_Ride.tns

Discussion Points and Possible Answers:

TI-Nspire Problem/Pages 1.2–1.4



Teacher Tip: *The function graph shown on page 1.2 of the TI-Nspire document is crafted piecewise out of several functions. On the TI-Nspire handheld, the graph takes a few moments to appear because of this complexity. Once the graph appears, the up/down arrows in the upper right part of the screen provide the means for moving the location of the roller coaster car, point x.*

1. The graph on page 1.2 represents a roller coaster at a state park. The polygon located at x = 0 represents the roller coaster car. The *x*-value and the slope of the tangent line (the first derivative, f'(x)) are calculated for each point on the curve.

   a. Click on the up or down arrow on the screen to move the car along the roller coaster and identify all the critical points.

   Students should notice seven critical points. The derivative is undefined where *x* = 1, *x* = 2, and *x* = 4, and zero where *x* = 6, *x* = 8.5, *x* = 12.25, and *x* = 15.

   b. List the critical points, explain <u>why</u> each of the points is a critical point, and use the first derivative test to prove the point is a local maximum, local minimum, or neither. Imagine yourself on the roller coaster. What happens to the rider at each of those points?

   Completed table is below and on the next page.

Critical point	***Reason why it is a critical point***	***Use the first derivative test to prove the critical point is a local maximum, local minimum, or neither.***	***Describe the ride at the critical point.***
x = 1	*f' is undefined*	The derivative does not change, so the point is neither a local maximum nor a local minimum.	The ride is just getting started and has leveled off a little.
x = 2	*f' is undefined*	The derivative does not change, so the point is neither a local maximum nor a local minimum.	The ride takes a sharp turn to head for the long stretch up.
x = 4	*f' is undefined*	The derivative changes from positive to negative; the point is a local maximum.	The ride reaches the cusp and then drops down suddenly.
x = 6	*f' is zero*	The derivative changes from negative to positive; the point is a local minimum.	The ride slows down and turns around to climb again.

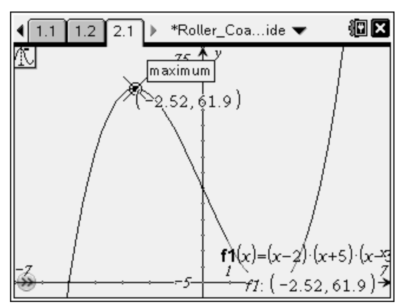
Critical point	***Reason why it is a critical point***	***Use the first derivative test to prove the critical point is a local maximum, local minimum, or neither.***	***Describe the ride at the critical point.***
x = 8.5	*f′ is zero*	The derivative changes from positive to negative; the point is a local maximum.	The ride reaches another height and turns to fall again.
x = 12.25	*f′ is zero*	The derivative changes from negative to positive; the point is a local minimum.	The ride slows down and turns around to climb again.
x = 15	*f′ is zero*	The derivative changes from positive to negative; the point is a local maximum.	The ride drops suddenly for the last time.

c. Complete the definition of the ***first derivative test*** below:

Suppose f is continuous at the critical point a:

- *If the first derivative f′ changes sign from* ***positive*** *to* ***negative*** *at a, then f(a) is* ***a local maximum.***
- *If the first derivative f′ changes sign from* ***negative*** *to* ***positive*** *at a, then f(a) is* ***a local minimum.***
- *If the first derivative f′ does not change sign at a, then f has* ***neither a local maximum nor a local minimum.***

TI-Nspire Problem/Page 2.1



2. a. Find the derivative function $f'(x)$ for the function $f(x) = (x - 2)(x + 5)(x - 3)$.

$f'(x) = 3x^2 - 19$

b. Fill in the table below for the given values of x.

x	***f′(x)***
–3	***8***
–2	***–7***
–1	***–16***
0	**–19**
1	**–16**
2	**–7**
3	**8**

c. Using the information from the table, speculate about the location of any local maxima or minima. Where are the local extrema?

Because the derivative changes from positive to negative between –3 and –2, students should guess there is a local maximum at about –2.5.

Where the derivative changes from negative to positive, between 2 and 3, there should be a local minimum.

d. Graph the function $f(x) = (x - 2)(x + 5)(x - 3)$ on page 2.1 to verify your answers above.

Local maximum: $x = -2.52$

Local minimum: $x = 2.52$

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify the critical points of a function given its graph.
- Why the local minima or maxima of a function occur at its critical points.
- Why not every critical point is a local minima or maxima.
- How the first derivative test can be used to determine if a critical point is a local minimum, local maximum, or neither.