

### Activity Overview

In this activity, students use a combination of algebra and graphing to discover the properties of logarithms. Given an equation such as  $b = a^2$  and a table of values for  $a$ , students define  $x = \log a$  and  $y = \log b$  and graph the resulting sets on the coordinate plane, graphically “taking the log of both sides.” Finding the equation of this graph and working backwards reveals an equation that expresses one of the properties of logarithms. Seeing a power function transformed into a linear function visually reinforces the effect of the log. Problems 1 and 2 cover the Power and Product Properties, respectively; Problem 3 covers the Quotient Property in addition to introducing log-log “paper”, a grid with a logarithmic scale on each axis.

### Topic: Exponential & Logarithmic Functions

- Derive algebraically the rules for products, quotients, and powers of logarithmic expressions.
- Verify graphically the rules for products, quotients, and powers of logarithmic expressions.

### Teacher Preparation and Notes

- This activity is appropriate for students in Algebra 2 or as a review for Precalculus. Prior to beginning this activity, students should be familiar with the definition of a logarithm and have experience with simple exponential equations of the form  $a^x=b$ , as well as the exponent rules.
- Before beginning the activity, students should transfer the Cabri™ Jr. file **LOGLOG** to their graphing calculators. Students should also clear all lists from the list editor by pressing  $\boxed{2\text{nd}} \boxed{[\text{MEM}]}$  and selecting **ClrAllLists**.
- To download the student worksheet and Cabri™ Jr. file, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “99606” in the keyword search box.

### Suggested Related Activities

To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.

- Exponential Reflections (TI-84 Plus family) — 12276
- Wrapping It All Up (TI-84 Plus family) — 4290

L2	L3	L4	4
6	0		-----
12	.30103		
18	.47712		
24	.60206		
30	.69897		
36	.77815		
42	.8451		

$L4 = \log(L2)$

This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus Silver Edition. It is also appropriate for use with the TI-83 Plus and TI-84 Plus but slight variances may be found within the directions.

#### Compatible Devices:

- TI-84 Plus Family

#### Software Application:

- Cabri™ Jr.

#### Associated Materials:

- PropertiesOfLogarithms\_Student.pdf
- PropertiesOfLogarithms\_Student.doc
- LOGLOG.8xv

Click [HERE](#) for Graphing Calculator Tutorials.



### Problem 1 – The Power Property of Logarithms

In this problem students will explore how to simplify the logarithm of a power, like  $\log a^2$ .

They will start by defining a new variable  $b = a^2$ , making a table of values for  $a$  and  $b$  using the calculator's **List** feature.

L1: 10 values of  $a$  within the logarithm domain

L2: values of  $b$  using the formula  $L1^2$  ( $\text{2nd}$   $[L1]$   $[x^2]$ )

Students will set up a scatter up with the settings given on the worksheet. They should make sure that all functions in the  $\text{Y=}$  screen have been turned off before viewing the plot.

Students may relate this graph to half of a parabola.

Now they will define two new variables,  $x$  and  $y$ , letting  $x = \log a$  and  $y = \log b$ .

L3 values of  $x$  using the formula  $\log(L1)$  ( $\text{LOG}$   $\text{2nd}$   $[L1]$ )

L4 values of  $y$  using the formula  $\log(L2)$  ( $\text{LOG}$   $\text{2nd}$   $[L2]$ )

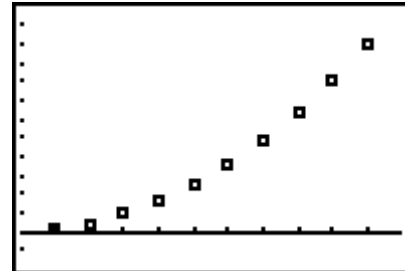
Now students will change **Plot1** to set up a scatter plot of with L3 as the Xlist and L4 as the Ylist.

Students should notice that the plot forms a straight line. Ask them what appears to be different about this plot as compared to the previous one (*spacing of points*). This will prepare them for Problem 3.

Students are directed to find the equation of a line through these points with the **LinReg** command. The command should appear on the Home screen as **LinReg(ax+b) L3, L4**.

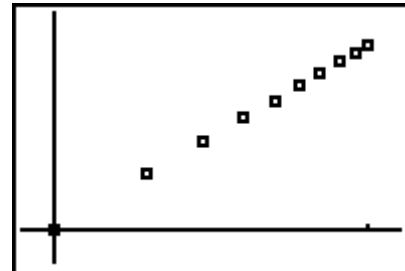
L1	L2	L3	2
1	1	-----	
4	16		
9	81		
16	256		
25	625		
36	1296		
49	2401		

$L2 = L1^2$



L2	L3	L4	4
1	0	-----	
4	.30103		
9	.47712		
16	.60206		
25	.69897		
36	.77815		
49	.8451		

$L4 = \log(L2)$



```
LinReg
y=ax+b
a=2
b=0
```



### If using Mathprint™ OS:

When entering the command for the regression equation, students can enter Y1 by either (1) press **[VARS]** **[>]** **[ENTER]** and select Y1 or (2) press **[ALPHA]** **[F4]** and select Y1.

```

CubicReg
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:Y1
Calculate

```

The equation that results is  $y = 2x$ .

The worksheet shows students how to rewrite this equation in terms of  $a$ , which gives an example of the Power Property of Logarithms. Be sure that students fully understand this property before moving onto Problem 2.

**Note:** When using the Regression commands, students do not need the Stat Diagnostics displayed. To turn this off, press **[2nd]** **[CATALOG]** **[D]** and select **DiagnosticOff**. Then press **[ENTER]** on the Home screen. This should be done before using **LinReg**.

```

CATALOG
DependAsk
DependAuto
det(
DiagnosticOff
DiagnosticOn
dim(
Disp

```

### If using Mathprint™ OS:

The Diagnostic feature can be turned off from the Mode screen. Students can press **[MODE]** and press enter on **OFF** next to **STATDIAGNOSTICS**. Then press **[2nd]** **[QUIT]** to return to the Home screen.

```

      TRACk†
MATHPRINT CLASSIC
M/D Un/d
ANSWERS: AUTO DEC FRAC
GOTO FORMAT GRAPH: NO YES
STATDIAGNOSTICS: OFF ON
SET CLOCK 01/02/34 1:14AM

```

### Problem 2 – The Product Property of Logarithms

Students will follow the process of Problem 1 to explore how to simplify the logarithm of a product, like  $\log 6a$ .

They will start by defining a new variable  $b = 6a$ .

Students need to clear all the data from **L2-L4**, but not from **L1**. To clear these lists, they can move to the top of a column and press **[CLEAR]** **[ENTER]**.

L1: 10 values of  $a$  within the logarithm domain

L2: values of  $b$  using the formula  $6 \cdot L1$  (**[6]** **[x]** **[2nd]** **[L1]**)

L1	L2	L3
1	6	2
2	12	-----
3	18	
4	24	
5	30	
6	36	
7	42	

L2 = 6 \* L1



Students will change **Plot1** to set up and view a scatter plot of L1 as the Xlist and L2 as the Ylist.

They are to make a prediction as to what the slope and y-intercept of the line is, and then use the **LinReg** command to find the actual values.

Now they will define two new variables, x and y, letting  $x = \log a$  and  $y = \log b$ .

L3 values of x using the formula **log(L1)** ((LOG) [2nd] [L1])

L4 values of y using the formula **log(L2)** ((LOG) [2nd] [L2])

After viewing the scatter plot with L3 as the Xlist and L4 as the Ylist, students should see that the data appears linear. They should also notice that the points form the same pattern as they did in Problem 1.

Using the **LinReg** command, students will find the equation of a line through these points. They should have found that the equation of the line was  $y = x + 0.778151$ .

$$10^{0.778151} \approx 6$$

The worksheet gives an opportunity for students to rewrite this equation in terms of a, which then gives an example of the Product Property of Logarithms. Be sure that students fully understand this property before moving onto Problem 3.

$$y = \log 6 + x$$

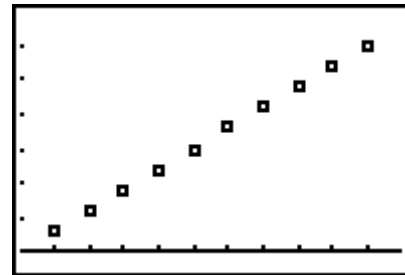
$$\log b = \log 6 + \log a$$

$$\log 6a = \log 6 + \log a$$

### Problem 3 – The Quotient Property of Logarithms

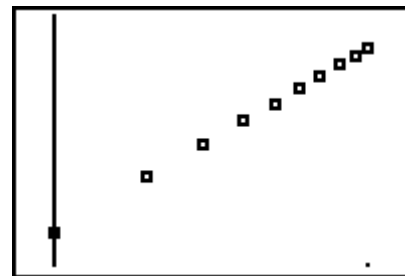
In this problem, students will use log-log “paper” to simplify the expression  $\log \frac{8}{a}$ .

To see an example of log-log paper, launch the **Cabri™ Jr** app and open the file **LOGLOG**. Students should see that the line of the grid resemble the spacing of the scatter plots of L3 vs. L4.

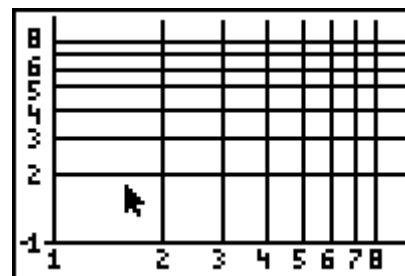


L2	L3	L4
6	0	-----
12	.30103	
18	.47712	
24	.60206	
30	.69897	
36	.77815	
42	.8451	

L4 = log(L2)



```
LinReg
y=ax+b
a=1
b=.7781512504
```

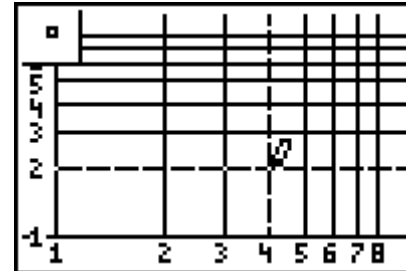




Explain to students that log-log paper is made by taking the log of each value of  $x$  and  $y$  and then renumbering the axes. They should note that the spaces between the lines are not equal. That is because the distance from 1 to 2 on a log scale is equal to  $\log 2$  units, and the distance from 1 to 3 is equal to  $\log 3$  units.

They will first define  $b = \frac{8}{a}$ .

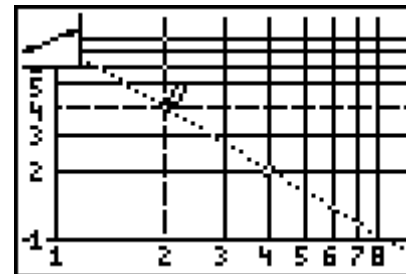
To plot the points from the function  $y = \frac{8}{x}$  on the log-log paper, students are to only use the points with whole number coordinates. There will be four points (1, 8), (2, 4), (4, 2), and (8, 1).



To plot a point, students need to press **WINDOW** for the **F2:Creation** menu and select **Point**. When the lines of the intersection are flashing, students should press **ENTER**.

They should see that the shape of the graph is linear.

To draw a line through the points, students need to press **WINDOW** for the **F2:Creation** menu and select **Line**. When the point is flashing, students should press **ENTER**, move to the next point and when it flashes press **ENTER**.



The equation of this line with respect to the linear scale (not the log scale) is  $y = 0.90309 - x$ .

$$10^{0.90309} \approx 8$$

The worksheet gives an opportunity for students to rewrite this equation in terms of  $a$ , which then gives an example of the Quotient Property of Logarithms.

$$\begin{aligned} y &= \log 8 - x \\ \log b &= \log 8 - \log a \\ \log \frac{8}{a} &= \log 8 - \log a \end{aligned}$$

To close the activity, explain to students that base 10 logarithms were used throughout this activity for convenience only. These properties apply to logarithms with any base.

### Solutions – Student Worksheet

- |                                     |  |
|-------------------------------------|--|
| 1. half of a parabola               | 8. 0.778151  |
| 2. straight line                    | 9. $\log(6)$   |
| 3. $y = 2x$                         | 10. $10^{0.778151} \approx 6$ , $\log(6) \approx 0.778151$   |
| 4. straight line                    | 11. log-log paper is made by taking the log of each value of $x$ and $y$ and then renumbering the axes |
| 5. Slope = 6 and $y$ -intercept = 0 | 12. straight line  |
| 6. $y = 6x$                         | 13. $\log(8)$  |
| 7. $y = x + 0.778151$               | 14. $10^{0.90309} \approx 8$ , $\log(8) \approx 0.90309$   |