Special Right Triangles

MATH NSPIRED

Math Objectives

- Students will identify the relationship between the lengths of the shorter and longer legs in a 30°-60°-90° right triangle.
- Students will determine the relationship between the shorter leg and the hypotenuse in a 30°-60°-90° right triangle.
- Students will identify the relationship between the lengths of the legs in a 45°-45°-90° right triangle.
- Students will determine the relationship between the legs and the hypotenuse in a 45°-45°-90° right triangle.
- Students will attend to precision (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- hypotenuse
- altitude
- right triangle

About the Lesson

- This lesson involves manipulating a special right triangle that is half of an equilateral triangle (the 30°-60°-90° triangle) and a special right triangle that is half of a square (the 45°-45°-90° triangle).
- As a result, students will:

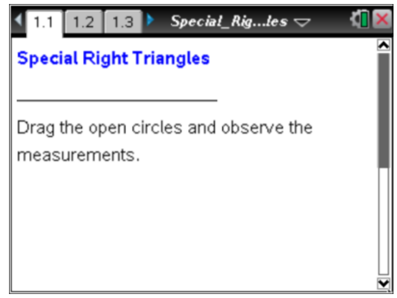
  Determine the relationships among the lengths of the sides of a 30°-60°-90° triangle and a 45°-45°-90° triangle.

TI-Nspire™ Navigator™

- Use Class Capture to formally assess students' understanding.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Special_Right_Triangles_Student.pdf
- Special_Right_Triangles_Student.doc

TI-Nspire document

- Special_Right_Triangles.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (👆) getting ready to grab the point. Press ctrl 🖐 to grab the point and close the hand (✊).

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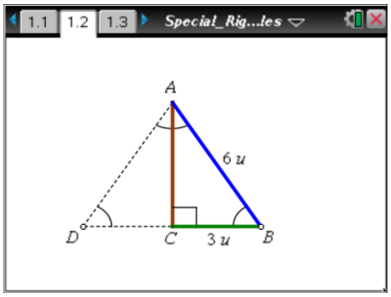

1. $\triangle ABD$ is an equilateral triangle. Drag point *B* or *D*.

   a. What kind of triangle is $\triangle ABC$? What are its angle measures? How do you know?

   <u>Answer:</u> ΔABC is a right triangle. The angles are 30°, 60°, and 90°. Since ΔABD is equilateral, $\angle ABC$ is 60°. Since $\overline{AC}$ is perpendicular to $\overline{DB}$ (as shown by the right angle mark), then $\overline{AC}$ is also the angle bisector of $\angle DAB$, making $\angle CAB$ one-half of 60°.

   b. What do you observe about *AB* and *CB*? Write an equation showing the relationship.

   <u>Answer:</u> *AB* always appears to be twice *CB*. Possible equations: $AB = 2 \times CB$; $CB = \frac{AB}{2}$; $\frac{AB}{CB} = 2$; $\frac{CB}{AB} = \frac{1}{2}$; or equivalent.

   c. Given the measures for *AB* and *CB*, how can the exact value of *AC* be calculated?

   <u>Answer:</u> $\overline{AC}$ is a leg of a right triangle, so its length can be calculated using the Pythagorean Theorem.

2. Drag point *B* to get the values of *CB* given in the table. Record the missing measures of *AB* and *AC* (use the Pythagorean Theorem to calculate and record exact values for *AC*). Write the ratio for the fourth column.

Answer:

AB (hypotenuse)	*CB* (shorter leg)	*AC* (longer leg)	$\frac{AC}{CB}$
4	2	$2\sqrt{3}$	$\sqrt{3}$ or $\frac{2\sqrt{3}}{2}$
6	3	$3\sqrt{3}$	$\sqrt{3}$ or $\frac{3\sqrt{3}}{3}$
8	4	$4\sqrt{3}$	$\sqrt{3}$ or $\frac{4\sqrt{3}}{4}$

Teacher Tip: Students may want to use a calculator and record decimal answers for the length of *AC* or the ratio. Be certain students understand that irrational numbers have decimal approximations that are not the exact value of the number. To be exact, the square root is still needed.

3. Examine the table from question 2.

a. What do you observe about *CB* and *AC?* Test your observation using another length of $\overline{CB}$.

Answer: *AC* always is *CB* multiplied by $\sqrt{3}$. Test answers will vary. Example: If *CB* is 5, then *AB* is 10 and *AC* is $5\sqrt{3}$.

b. Write an equation showing the relationship between *CB* and *AC* from your observations.

Sample Answers: Possible equations: $AC = CB \times \sqrt{3}$; $\frac{AC}{CB} = \sqrt{3}$; or equivalent

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

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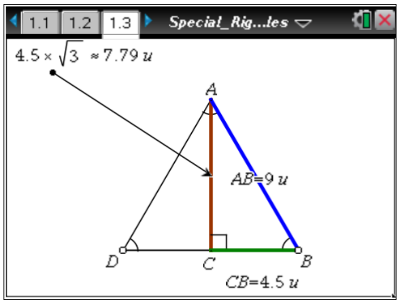

4. Grab and drag point *B* or *D*.

    a. What do you observe about the calculation and the measure of *AC*? Does this confirm or disprove your equation in question 3b?

    <u>Answer:</u> The calculation shows $CB \times \sqrt{3}$, which matches the measure of *AC* for every location of *B* and *D*. Answers about confirm/disprove will vary based on answers to question 3b. Example: My equation was $AC = CB \times \sqrt{3}$, so the measurement confirms my equation.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

> **Teacher Tip:** If students gave any correct equations other than $AC = CB \times \sqrt{3}$, discuss equivalence among the various equations to address this question.

b. Describe the special right triangle in this investigation and express relationships that always exist among the shorter leg, longer leg, and hypotenuse.

<u>Answer:</u> In a 30°-60°-90° right triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

> **Teacher Tip:** Lead a discussion about how the ratios can be used to find lengths in a triangle with only some lengths given.

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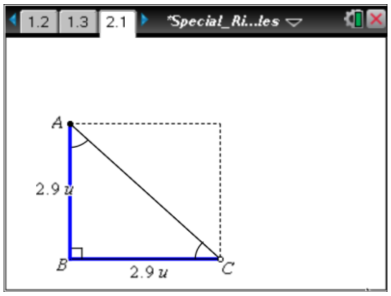

5. $\triangle ABC$ is half of a square. Drag point *C*.

    a. What kind of triangle is $\triangle ABC$? What are its angle measures? How do you know?

    <u>Answer:</u> ΔABC is an isosceles right triangle. Its angles are 45°, 45°, and 90°. $\measuredangle ABC$ is a right angle because it is an angle of the square. Since $\overline{AC}$ is a diagonal of the square, it bisects both angles of the square that it intersects, forming 45° angles.

b. What do you observe about *AB* and *CB?* Write an equation showing the relationship.

Answer: *AB* and *CB* remain congruent for all positions of *B* or *C*. *AB* = *CB*.

6. Drag point *C* to get the values of *CB* given in the table. Record the missing measures of *AB* and *AC* (use the Pythagorean Theorem to calculate and record exact values for *AC*). Write the ratio for the fourth column.

Answer:

AB (leg)	*CB* (leg)	*AC* (hypotenuse)	$\frac{AC}{CB}$
2	2	$2\sqrt{2}$	$\sqrt{2}$ or $\frac{2\sqrt{2}}{2}$
3	3	$3\sqrt{2}$	$\sqrt{2}$ or $\frac{3\sqrt{2}}{3}$
4	4	$4\sqrt{2}$	$\sqrt{2}$ or $\frac{4\sqrt{2}}{4}$

7. Examine the table in question 6.

a. What do you observe about *CB* and *AC?* Test your observation using another length of $\overline{CB}$.

Answer: *AC* is the same as *CB* multiplied by $\sqrt{2}$. Test answers will vary. Example: if *CB* is 5, then *AB* is 5 and *AC* is $5\sqrt{2}$.

b. Write an equation showing the relationship between *CB* and *AC* from your observations.

Sample Answers: $AC = CB \times \sqrt{2}$; $\frac{AC}{CB} = \sqrt{2}$; or equivalent

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

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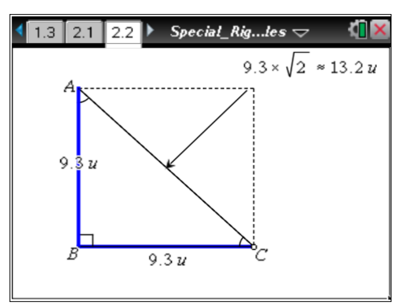
8. Drag point *C*.



a. What do you observe about the calculation and the measure of *AC?* Does this confirm or disprove your equation in question 7b?

Answer: The calculation shows $AC = CB \times \sqrt{2}$ for every location of *B* and *C*. Answers about confirm/disprove will vary based on answers to question 7b.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 4 at the end of this lesson.

b. Describe the special right triangle in this investigation and express relationships that always exist among the legs and hypotenuse.

Answer: In a 45°-45°-90° (isosceles) right triangle, the legs are congruent, and the hypotenuse is $\sqrt{2}$ times as long as a leg.

Teacher Tip: Lead a discussion about how the ratios can be used to find lengths in a triangle with only some lengths given.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to determine missing lengths in a 30°-60°-90° triangle using the ratios among the sides.
- How to determine missing lengths in a 45°-45°-90° triangle using the ratios among the sides.

Assessment

1. The perimeter of a square courtyard is 20 feet. How long would a diagonal path across the courtyard be?

   Answer: $5\sqrt{2}$ feet

2. A stencil in the shape of an equilateral triangle has 1-inch sides. What is the altitude of the stencil?

   Answer: $\frac{\sqrt{3}}{2}$ inches

3. Erin needs to draw an equilateral triangle that is 10.4 decimeters tall to fit on a poster. What will be the length of the sides of the triangle?

   Answer: About 12 decimeter each

4. A square piece of paper is folded along a diagonal of the square to form a triangle. The creased side is about 8.5 inches long. What are the dimensions of the square?

   Answer: About 6 inches by 6 inches

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Note 1

Question 3b, *Quick Poll*: Once students have answered question 3b, send the following Open Response Quick Poll:

If *CB* is 8, then *AB* is 16 and *AC* is _____.

Answer: $AC = 8\sqrt{3}$

Note 2

Question 4a, *Class Capture*: Take a Class Capture as students are moving points *B* and *D* so that they can see and verify the relationship that exists for the longer leg, shorter leg, and hypotenuse in all 30°-60°-90° triangles.

Note 3

Question 7b, *Quick Poll*: Once students have answered question 7b, send the following Open Response Quick Poll:

If *CB* is 8, then *AB* is 8 and *AC* is _____.

Answer: $AC = 8\sqrt{2}$

Note 4

Question 8a, *Class Capture*: Take a Class Capture as students are moving point *C* so that they can see and verify the relationship that exists for the legs and hypotenuse in all 45°-45°-90° triangles.