Modelling Compound Interest using Recursion

Student Activity

7   8   9   10   **11   12**

TI-Nspire   Investigation   Student   30 min

Introduction

Recursive techniques are very useful for finance problems that use a repetitive process based on previous answers (iterations). i.e. compounding interest or reducing loans at set rates.

You will have done simple arithmetic recursive exercises before without knowing what the term recursive meant.

Introductory tasks

In a **Calculator** page, enter 12 and press enter.

On the next line press +3. As no answer exists on this new line it will assume you wish to *add 3* to the previous answer (i.e. 12) and will automatically paste **Ans** prior to the +3 and when you press enter it will change the **Ans** to the 12 and display the new result. Continue pressing enter to get further iterations. The list of numbers generated this way is called a sequence.

This is sometimes written as V_0=12, V_{n+1}=V_n+3, i.e. the initial value V_0 =12, the next value (V_{n+1})= previous value (V_n) +3.

In a **Calculator** page, enter 20 and press enter.

On the next line press ×4-1. As no answer exists on this new line it will assume you wish to *multiply by 4 and subtract 1* to the previous answer (i.e. 20) and will automatically paste **Ans** prior to the ×4-1 and when you press enter it will change the **Ans** to the 20 and display the new result. Continue pressing enter to get further iterations. The list of numbers generated this way is called a sequence.

This is sometimes written as V_0=20, V_{n+1}=V_n×4-1, i.e. the initial value V_0 =20, the next value (V_{n+1})= previous value (V_n) ×4-1.

In Finance questions, using recursive techniques, the V_n is the value of the investment (answer at each line) at the end of each compounding period. The initial value entered will be P (the principal). In the simple example above it would be the 12. Instead of "+3" you would enter "$\times\left(1+\dfrac{r/c}{100}\right)$" where r/c is the annual interest rate (r) divided by the <u>compounding periods per year</u> (c).

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TEXAS INSTRUMENTS

Example 1:

Determine the amount of money accumulated after 4 years if $10 000 is invested at an interest rate of 6.25% per annum, compounded <u>annually</u>.

Note that period 4 (i.e. after 4 compounding periods) you are referring to the 5th term as term 1 (period 0) is the initial value.

For simplicity, the value inside the brackets $\left(1+\dfrac{6.25/1}{100}\right)$, is often computed first. i.e. 1.0625

This problem can be also written as $V_0=10000$, $V_{n+1}= V_n\times 1.0625$

In a **Calculator** page enter the initial value (Principal) and press enter. i.e. 10000

On the 2nd line press ***(1+6.25/100) or *1.0625**, then enter. It will use the previous answer to complete this computation. Continue to press enter to generate the required iterations.

1.1	1.2	1.3	▶ *CI – student	DEG
10000				10000.
Ans· 1.0625				
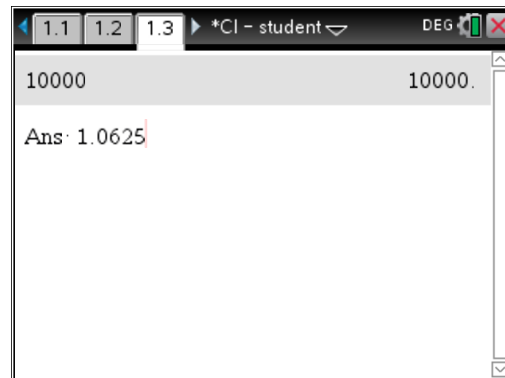
What is the amount of money accumulated after 4 years compounded annually?

Answer:

Note that you are looking for the 5th term as the 1st term is the initial value.

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TEXAS INSTRUMENTS

Example 2:

Maria invests $3500 at an interest rate of 6.0% p.a., compounded <u>monthly</u>. After how many months will her investment first exceed $3600?

This problem is a little more complicated than the first example as it is compounded <u>monthly.</u> Be careful when working out the value you will use to multiply your answers each time as the compounds occur every month.

Hint: substitute in the values for r & c in $"\times\left(1+\dfrac{r/c}{100}\right)"$ to find the value you will use to multiply your answers each time.

In a **Calculator** page enter the initial value (Principal) and press enter. i.e. 3500.

On the 2nd line type in $\times$ and your computed value, then enter. It will use the previous answer to complete this computation. Continue to press enter to generate the required iterations.
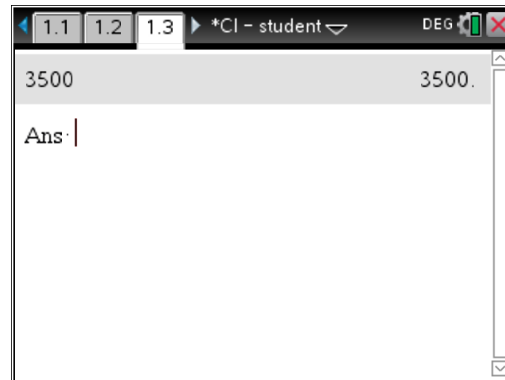
After how many months will her investment first exceed $3600?

Answer:

Using a spreadsheet to show recursive technique for Example 1 above

Recursive approach using a spreadsheet.

Insert a Lists & Spreadsheet page.

Enter list names **period** and **amount.**

Enter compounding periods in column A. Start with 0 for the initial start period equalling the initial investment and enter up to 12 periods. Enter the initial investment value in cell b1.

In cell b2 enter = b1*(1.0625), enter. Fill down ([menu]>**Data>Fill**).

What is the amount after 4 years?

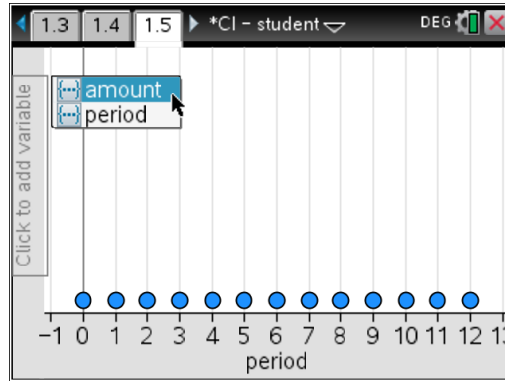Amount after 4 years is ………….

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Insert a **Data & Statistics** page and the data created in the spreadsheet. Use *period* on the x-axis and *amount* on the y-axis.

Discuss the shape of the curve.

Does it appear to increasing at an even amount each year or increasing at a higher rate as the periods get larger?

Use the spreadsheet to show the results for Example 2
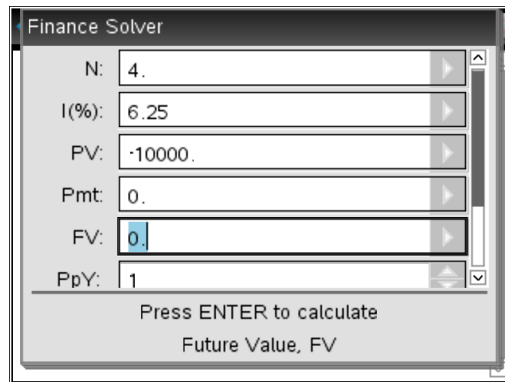
Using TVM Solver

Access on a **Calculator** page using [menu]>**Finance>Finance Solver**

N is the number of payments or compounds for the term of the investment.

I is the interest rate (p.a)

PV (Present Value) is entered as negative as it has been loaned to the financial institution.

Ppy and Cpy will be the same and is the number of compounds per year.

Note that the Finance Solver can only be used for compounding/reducing balance examples and not simple interest based problems.

Place cursor on the unknown value (in this case the Future value, FV), and press enter to evaluate. This is the amount the investment will be worth at the end of the investment period.

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