Teacher Notes

Activity 15

Approximating Integrals with Riemann Sums

Objectives

- Calculate Riemann sums
- Demonstrate when Riemann sums will over-approximate or under-approximate a definite integral
- Observe the convergence of Riemann sums as the number of subintervals gets larger

Materials

- TI-84 Plus / TI-83 Plus
- NUMINT program

Teaching Time

- 50 minutes

Abstract

In this activity, students will calculate and analyze Riemann sums. They will draw rectangles whose areas correspond to terms of Riemann sums and observe the convergence of left-hand and right-hand Riemann sums by using graphing handhelds to automate the production of Riemann sums with regular partitions having a large number of subintervals.

Advance Preparation

Load the program **NUMINT** on the students' graphing handhelds. The program is available in Appendix B at the back of this book or from the TI Web site: **education.ti.com**.

Management Tips and Hints

Prerequisites

Students should be able to:

- graph functions, generate tables, and use the **TBLSET Menu** to change the starting value and increment in a table.
- select and execute a program from the **PRGM Menu**.

This activity is most appropriate as an introduction to the definite integral, although students must be familiar with the notation for an integral.

Evidence of Learning

- Students can calculate left- and right-hand Riemann sums without using a calculator program.

- Students can predict whether a left or right Riemann sum will under- or over-approximate a definite integral, when the integrand is monotonic over the interval of integration.
- Students can predict whether a left- or right-hand Riemann will get larger or smaller as partition size increases.

Common Student Errors/Misconceptions

- When doing Riemann sums by hand, some students will forget to multiply the function outputs by the width of each subinterval.
- Students may include an extra term at the end of a left-hand sum or at the beginning of a right-hand sum.
- Teachers should caution students about the dangers involved in entering a large number for the number of subintervals at the start of the process. The limited precision of the graphing handheld could cause unreliable results. Students should increase the number of subintervals gradually to get a feel for the value to which Riemann sums might converge.
- It can take a couple of minutes for the graphing handheld program to find a sum with 1,024 subintervals.

Teaching Hints

The natural follow-up to this activity is defining the definite integral. Prior to the advent of inexpensive programmable calculators, it was difficult to convince students that a Riemann sum would converge to a number. Now, it is easy to generate Riemann sums with hundreds of terms.

Extensions

Repeat the activity using the midpoint rule. Have students produce a table of inputs and outputs (if 2 subintervals are used over the interval $x = 1$ to $x = 5$, the inputs should be $x = 2$ and $x = 4$). Set **TblStart** to 2 and **ΔTbl** to 2. Then change **ΔTbl** to 0.5. Finally, use the graphing handheld program, and add an extra column to the table from Question **13**.

Investigate a function that is increasing over the interval of integration, such as

$$\int_0^{\frac{\pi}{2}} \sin(x)\,dx$$

Ask students to predict which method, left-hand sum or right-hand sum, will over-approximate the integral. Alternatively, ask students to produce a function *g* for which the left-hand sum will under-approximate

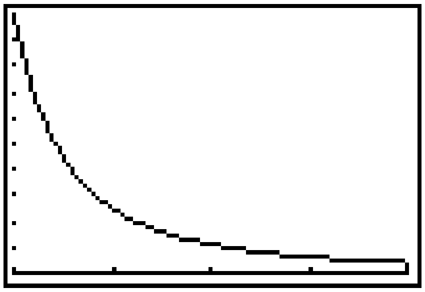
$$\int_1^5 g(x)\,dx$$

Repeat the activity using the trapezoid rule. Demonstrate that the trapezoid rule is equivalent to averaging the left-hand and right-hand sums. The **NUMINT** program leaves the most recently calculated sums in the list variable, **L6**. **L6**(1) is the left-hand sum, **L6**(2) is the right-hand sum, **L6**(3) is the midpoint sum, and **L6**(4) is the trapezoid sum. You could calculate (**L6**(1) + **L6**(2))/2 and compare the result with **L6**(4).

After students know how to evaluate a definite integral using the Fundamental Theorem of Calculus (so that the exact value of an integral is known), return to the **NUMINT** program, and use the values in **L6** to explore the errors from using the various approximations.

Activity Solutions

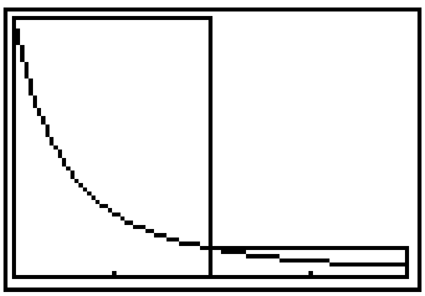
1.



2.

X	Y1(x)
1	1
3	0.11111

3.



4.

Rectangle from	Area
1 to 3	2
3 to 5	0.22222

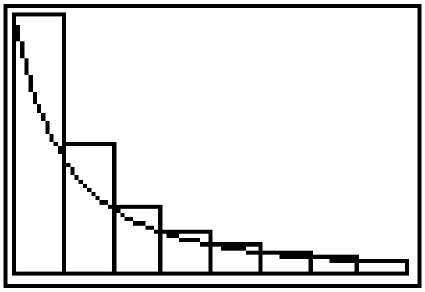
5. 2.22222

6. Larger; each rectangle extends above the graph of **Y1**. Thus, each includes additional area that is not under the graph of **Y1**.

7.

X	Y1(x)
1	1
1.5	0.44444
2	0.25
2.5	0.16
3	0.11111
3.5	0.08163
4	0.0625
4.5	0.04938

8.



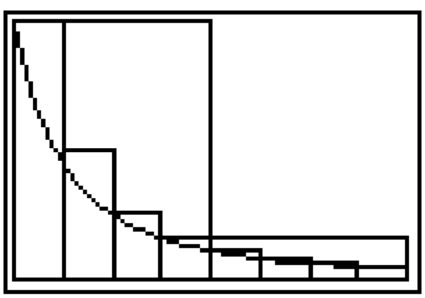
9.

Rectangle from	**Area**
1 to 1.5	0.5
1.5 to 2	0.22222
2 to 2.5	0.125
2.5 to 3	0.08
3 to 3.5	0.05556
3.5 to 4	0.04082
4 to 4.5	0.03125
4.5 to 5	0.02469

10. 1.07954

11. Larger

12. The sum of the 8 areas is smaller than the sum of the 2 areas. The screenshot shown supports this conclusion.



13.

Number of Subintervals	Left-Hand Riemann Sum	Right-Hand Riemann Sum
8	1.0795	0.5995
16	0.9302	0.6902
32	0.8626	0.7426
64	0.8306	0.7706
128	0.8152	0.7852
256	0.8075	0.7925
512	0.8038	0.7963
1024	0.8019	0.7981

14. Yes; 0.8

15. Smaller

16. As the number of rectangles increases, there is more of the area under the curve accounted for using the rectangles. Thus, the sums become larger.

17. Yes; 0.8