Pythagoras – Episode 2

Teacher Notes & Answers

7 8 **9** 10 11 12

TI-Nspire Investigation Student 30 min

Introduction

Congratulations, arriving here means you have completed the first clue. What did we learn and what do we need to know about history to understand why the Pythagorean Circle were so concerned about their new theroem?

The Pythagorean Circle believed numbers were the basis of reality, they were whole and harmonious. All things could be reduced to number relations, including ratios of whole numbers, numbers were rational and applied to practical situations. Upon using the formula, the Pythagorean Circle realised that when **you pen these** numbers the results did not align with their beliefs. In a world where 'zero' was yet to be invented, the notion of a number that was not finite created an enormous amount of angst, confusion and potential ridicule. The quest to find answers to some of the simplest triangles was proving problematic.

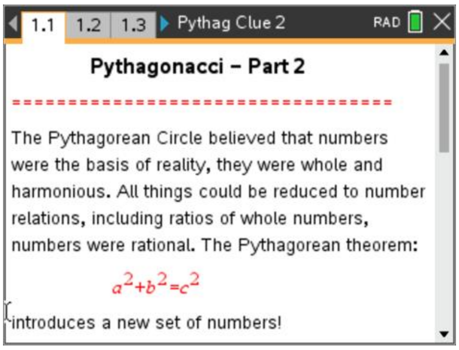
Your quest continues, to succeed you must first understand the implications of the theorem!

Note: you pen these … is an anagram of 'hypotenuse' which of course is the side of the triangle that generates irrational numbers, a number set that did not fit the ideal of the Pythagorean Circle. This is one of subtle clues.

Pythagoras

Open the TI-Nspire file: **Pythag Clue 2**

A series of triangles can be observed on page 1.2. Use the toggle



Clue 2:

Scroll through the collection of triangles. You will notice that some of the triangles appear as a different colour. Your task is to determine what is different (other than the colour), about these triangles.

Question: 1.

Scroll through the triangles using the slider/toggle:

a) Test each of the triangles on page 1.2 against the Pythagorean theorem.

Answers:

Triangle 1: 3, 4, 5 (Pythagorean Triple – Integer side lengths)

Triangle 2: 5, 12, 13 (Pythagorean Triple – Integer side lengths)

Triangle 3: 6, 8, 10 (Multiple of a Pythagorean Triple – Integer side lengths)

Triangle 4: 7.5, 10, 12.5 (Multiple of a Pythagorean Triple –Rational side lengths)

Triangle 5: 8, 15, 17 (Pythagorean Triple – Integer side lengths, not a multiple, clue: 17 is prime)

Triangle 6: 9, 12, 15 (Multiple of a Pythagorean Triple – All rational side lengths)

Triangle 7: 10, 12, $\sqrt{244}$ (Hypotenuse is irrational!) – This triangle is blue

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Triangle 8: 12, 13, $\sqrt{313}$ (Hypotenuse is irrational!) – This triangle is blue

Triangle 9: 10, 10, $\sqrt{200}$ (Hypotenuse is irrational!) – This triangle is blue

b) Aside from the colour, what makes triangles 7, 8 and 9 particularly special?

Answer: The hypotenuse is an irrational number, something the Pythagorean Circle would not have liked!

The Pythagorean Circle needed tools to deal with the special cases. The Babylonians (1500BC) established a technique for computing the square-root of a number by hand.

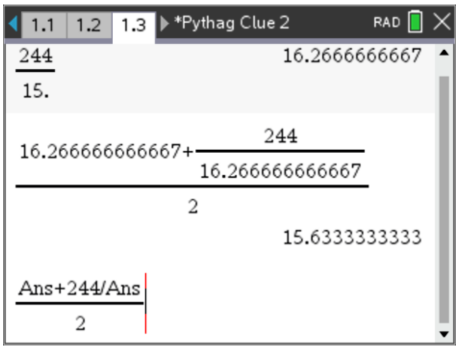
Suppose you want to calculate the square-root of 244. Start with an estimate, let's say 15, since $15^2 = 225$.

Step 1: Calculate $244 \div 15 \approx 16.2\dot{6}$ [Corrected value]

Step 2: Average the **estimate** (15) and the **corrected value**.

$(15 + 16.2\dot{6}) \div 2 \approx 15.6\dot{3}$

Step 3: Repeat Steps 1 & 2 using the result from Step 2 as the new estimate. [See opposite]

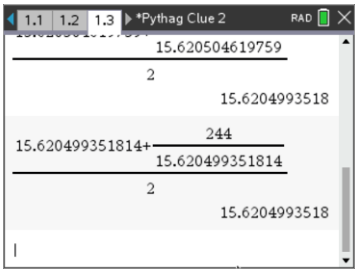


Question: 2.

Use 5 iterations of the Babylonian technique to establish an approximate square-root for 244 using 15 as the starting estimate.

Answer: The calculations shown opposite represent 5 iterations of the algorithm $\sqrt{244} \approx 15.620499351814$

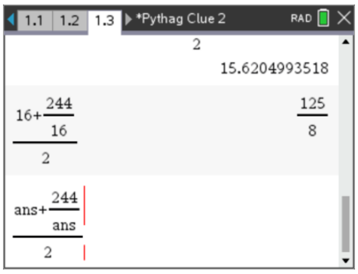
A remarkably accurate result, remembering of course that this has, ironically, been computed using a calculator, the by-hand computations would take much longer!



Question: 3.

Use 5 iterations of the Babylonian technique to establish an approximate square-root for 244 using 16 as the starting estimate.

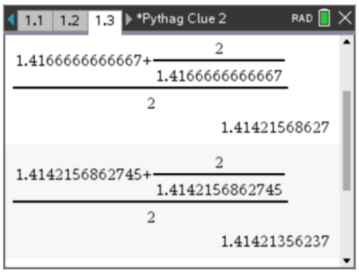
Answer: The calculations shown opposite illustrate the best way to set up the calculations. Repeating this process 5 times will result in a 'same' answer to the previous question. Correct to 12 decimal places!



Question: 4.

Use 3 iterations of the Babylonian technique to establish an approximate square-root for 2 using an initial guess of 1.5.

Answer: The calculations shown opposite represent 3 iterations of the algorithm $\sqrt{2} \approx 1.4142135623747$, again, remarkably close!



Question: 5.

Use 3 iterations of the Babylonian technique to establish an approximate square-root for 200 using an initial guess of 14.

Answer: The calculations shown opposite represent 3 iterations of the algorithm $\sqrt{200} \approx 14.142135623731$, again, remarkably close!

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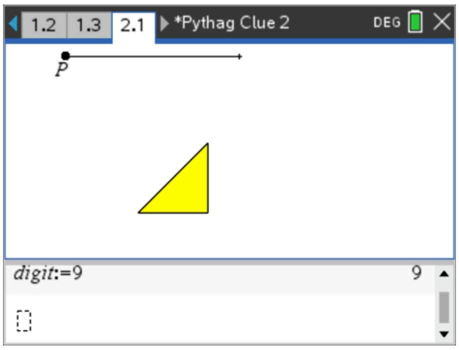


Navigate to problem 2, Page 1 (2.1) where the next clue is located.

To unlock the next clue, determine the last digit, that is the least significant in the multiplication of:

$$14.142135623731 \times 14.142135623731$$

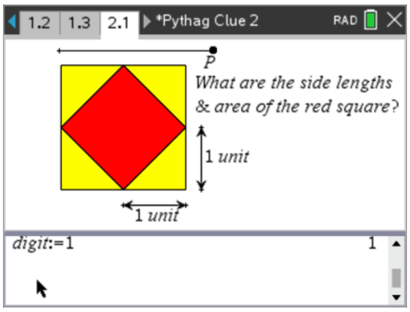
Once you have determined this quantity, store your answer in 'digit' as shown at the base of the screen (Calculator application).

To navigate to the base of the screen you can either click in the application or press [ctrl] + [tab] to switch application focus.

Once you have stored your result, drag point P slowly along the line.

Note: The digit is not a 9, which is why your next clue is not displayed.

Once you have the next clue, think carefully about your answer. You will need to explain your answer to your teacher. If your teacher is satisfied with your answer, they will provide you with the next document. Good luck.



Answer: Getting to the next clue:
The least significant digit in the calculation is 1. If students attempt to compute: 14.142135623731^2 on the calculator, it will display the result "2." indicating it has run out of decimal places and simply approximated the result.
The purpose of the question is to highlight that no matter how many decimal places we use, the product of the two numbers will not be 'exactly' 2, which of course is why surds are often used.
If students have the incorrect answer for the 'digit', (digit = 1), they will be able to see the animation but not the question.

Question: "What are the side lengths and area of the red square?

Side lengths = $\sqrt{2}$
Area of the square is = 2

Conundrum: If we approximate the side length (hypotenuse) the area or the square will not be exactly 2, we see this from the previous multiplication problem.
If we determine the area of the square using other means (overall area minus the area of the triangles, then the area of the square is exactly 2.
Is the problem with the number: $\sqrt{2}$ or is it with the theorem?
How do you explain a number that can not be written in its entirety?

It is important that students realise the difficulties this would have created for the members of the Pythagorean Circle. At this point in time, they were completely unaware of irrational numbers. The Babylonian algorithm helps find the approximate square-root, but does not tell us that the 'exact' result forms part of a 'new number system'.

There is no indication that the Pythagorean Circle understood 'irrational numbers'. The Greeks in the 5th Century CE are credited for identifying such numbers. In the interim, the Pythagorean Circle and other mathematicians explored ways to arrive at these *challenging* numbers.

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