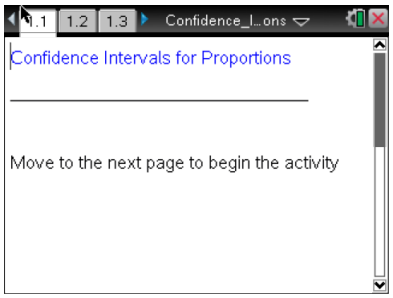
Confidence Intervals for Proportions
Student Activity

Name _____

Class _____

Open the TI-Nspire document *Confidence Intervals_for_ Proportions.tns.*



What do the results of a survey of a random sample of people tell you about the population? This activity investigates what a confidence interval is and the nature of conclusions that can be inferred from one.

If 40% of the people polled from a random sample support a stance, does this indicate that exactly 40% of the population from which the sample was draw will support the stance? Statisticians use a **confidence interval** to describe how to make an inference about a population from a sample.

Move to pages 1.2 and 1.3.

Press ctrl ▶ and ctrl ◀ to navigate the lesson.

Follow your teacher's directions to seed the random number generator.

Move to page 1.4.

A coffee house was interested in whether their early morning customers were mostly men or women. A survey indicated that, on average, the total number of customers was 62 per day for that time period.

1. Why might the company care about whether their customers were male or female?

2. Click the arrow on Page 1.4 to display the results of the survey. The men are marked with a blue dot and women with a pink x. What proportion of the customers in your sample was male?

Tip: Count as accurately as possible, but do not worry about missing one or two – the key idea is to determine about what proportion of men is in the sample**.** Be sure to record this value; it will be important throughout the activity.

The value you found is called the *observed sample proportion* and is often represented by the symbol $\hat{p}$, called '*p*-hat.' The question the owners want answered is what does this observed outcome— the proportion of males in the sample—indicate about the proportion of males in the population as a whole. The sample proportion will be important in the final steps of the activity, but for now, just note it as the answer to Question 2.

Move to page 2.1.

Samples vary, so before any statement can be made about the population, it is necessary to look carefully at what might happen when you know what the constraints are. The following questions explore how sampling variability can affect situations in which you know the population proportions (the "truth").

3. Suppose the proportions for males and females who frequented the coffee house were the same, 0.5.
   a. If you sample 62 customers, how many males would you expect in your sample? Do you think your expected number will be exactly correct? Explain your reasoning.

   b. Click the arrow on the left to display the proportion of men in a sample of 62 customers. What proportion of the customers were males?

Tip: Moving the cursor over a data point will display the exact proportion represented by that point.

   c. Click the arrow five more times. Write a statement describing the results and what they mean in terms of the proportion of men likely to be found in samples drawn from a population in which exactly 50% are male.

4. Continue to generate samples until you have 100 samples of size 62 from this 50-50 population. (Note that after the tenth sample, the results are displayed in sets of ten samples.)
   a. Describe the distribution.

   b. Based on your experience with these 100 samples, if you thought a population consisted of 50% men and 50% women, would you be surprised if the proportion of men in a sample of 62 people was 0.55? 0.20? Explain your reasoning in each case.

   c. Recall your observed proportion of men from Question 2. Would it seem likely that this sample proportion came from a population where the proportion of men was exactly 0.5? Why or why not?

Move to page 3.1.

The top arrow on the left of Page 3.1 allows you to set the proportion of men, **p**, for a given population. The bottom arrow on the left will generate samples of size 62 and display the proportion of men in each sample. After the first ten samples, the results are displayed in sets of ten until you have 100 samples. The arrow on the right will erase the distribution and allow you to start the simulation over.

5. Use the top arrow on the left to set the population proportion, ***p***, to 0.75. Use the bottom arrow on the left to generate a simulated sampling distribution for that proportion.
   a. Write a statement describing the results and what they mean in terms of the number of men in samples drawn from a population known to contain exactly 75% males.

   b. Recall your observed proportion of men from Question 2. Would it seem likely that this sample proportion came from a population where the proportion of men was exactly 0.75? Why or why not?

   c. Use the reset arrow on the right to erase the distribution. Using the same population proportion, ***p***, generate another simulated sampling distribution of the sample proportions of males. How will your answers to Questions 4a and b change for this new distribution?

Follow directions from your teacher to answer Question 6.

6. Click the top arrow to choose a population proportion (this might be assigned by your teacher). Generate 100 samples.
   a. Describe the distribution of the proportion of men in your 100 samples. In particular, what proportions seem typically to occur?

   b. Recall the observed proportion of males you found in Question 2. Does it seem likely that observed proportion could have come from the population you used in this simulation? Explain your reasoning.

Move to page 4.1.

Click the top arrow on the right to choose the observed sample proportion of men you generated on Page 1.4. The display will show a vertical line marking that observed proportion.
The top arrow on the left allows you to choose a given population proportion of men, and the display shows a simulated sampling distribution of the proportion of men based on 100 samples of size 62 drawn from that population, just as you saw in Question 6.

7. Choose a population proportion and use the yes/no arrow to indicate whether it seems likely the observed sample proportion of men, represented by the vertical line, could have come from that chosen population. Note that when you choose yes, the population proportion is plotted on the horizontal axis, **possible populations**, in the bottom panel.

   a. Write down the population proportions for which you chose "yes." Continue to choose different population proportions until you have marked and recorded an interval that seems to contain all reasonable "Yeses." Describe your interval.

   c. Use the reset arrow under $\hat{p}$ to simulate a new sampling distribution of proportions from samples of size 62 with the same population proportion. Repeat the directions for Question 7a to find a set of populations that might have produced the sample with the observed proportion of men. How does the new interval compare to the one you found in Question 7a?

The interval you found is called a *confidence interval* for a proportion. A confidence interval is an estimated range of values that is likely to contain an unknown population parameter, in this case proportion.

8. Suppose six months later the coffee house surveyed their customers on a randomly chosen day of the week. Again, the number of customers was 62. Return to Page 1.4, and generate a new sample.
   a. Record the proportion of men in the sample, return to Page 4.1, and find a confidence interval for the population proportion based on your new sample proportion.

   b. Write a few sentences to explain to someone who was not in class how to interpret your confidence interval.