Math Objectives

- Students will be able to input their data into matrices and perform calculations on them.
- Students will be able to interpret the results of their calculations.
- Students will utilize different methods to check the results of their calculations and to verify their solutions.
- Students will create a coherent representation for a problem (CCSS Mathematical Practice).
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).

Vocabulary

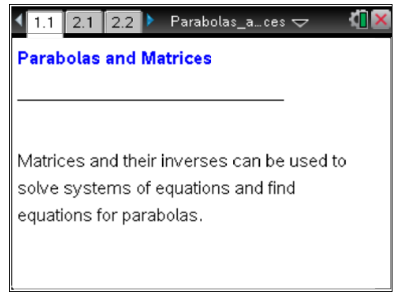
- identity matrix
- inverse matrix
- matrix
- matrix equation
- parabola
- system of equations

About the Lesson

- This lesson involves using matrices to solve a system of three simultaneous linear equations.
- As a result, students will:
  - Use matrices to write an equation for a parabola that passes through three given points.
  - Input their own matrices to solve problems that are similar to those outlined above.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Parabolas_and_Matrices_Student.pdf

Parabolas_and_Matrices_Student.doc

TI-Nspire document

Parabolas_and_Matrices.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Teacher Tip: Prior to beginning this activity, review with students how to organize information into matrices. This activity does not teach students the process used to set up and/or multiply matrices; rather it provides some examples of the types of problems that can be solved using inverse matrices with TI-Nspire technology. Students should already know how to multiply matrices by hand.

Teacher Tip: This problem helps students use inverse matrices to solve a system of equations. When students are translating the system to a matrix equation, be sure that they include all appropriate negative signs in the coefficient and/or constant matrices.

Let's review how you can use matrices to help you solve systems of equations.

First, write the system as a matrix equation of the form $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Then, multiply both sides of the equation by the inverse of the coefficient matrix, as shown to the right. (Be careful with the order in which you multiply the matrices!) Let's try an example.

$$AX = B$$

$$A^{-1} \cdot AX = A^{-1} \cdot B$$
$$X = A^{-1} \cdot B$$

1. Write this system as a matrix equation. In the 3 X 3 coefficient matrix, enter the coefficients of all three equations. In the 3 X 1 constant matrix, enter the three values of the constants.

Answer: $\begin{aligned} x - 2y + z &= 7 \\ 3x - 5y + z &= 14 \\ 2x - 2y - z &= 3 \end{aligned} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ 2 & -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \\ 3 \end{bmatrix}.$

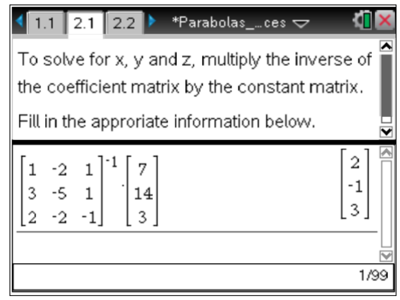
2. To solve for *x, y*, and *z*, multiply both sides of the equation above by the inverse matrix and simplify. (We will use the functionality of the TI-Nspire to obtain the inverse matrix.) Fill in the appropriate information below.

Answer:

$$\begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ 2 & -2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ 2 & -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ 2 & -2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ 14 \\ 3 \end{bmatrix}$$

Teacher Tip: If students are not familiar with using TI-Nspire to obtain and utilize an inverse matrix, you might want to progress a bit slowly through the next couple of questions.

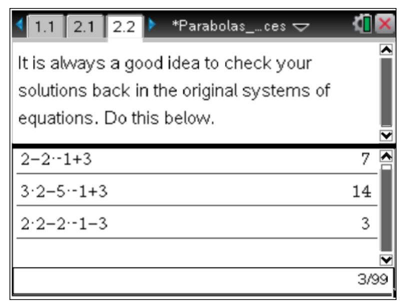
Move to page 2.1.



3. On Page 2.1, enter only the expression from the right side of the equation above, and press enter. Press tab to move from one entry to the next. In the spaces provided below, copy what you entered on page 2.1 and also fill in the solution matrix.

Answer: $\begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ 2 & -2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7 \\ 14 \\ 3 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Move to page 2.2.



4. Check your solutions in the original system of equations. Show your work below.

Sample Answers:

Given: $x - 2y + z = 7$ substitute x = 2, y = -1, z = 3 $2 - 2(-1) + 3 = 2 + 2 + 3 = 7$

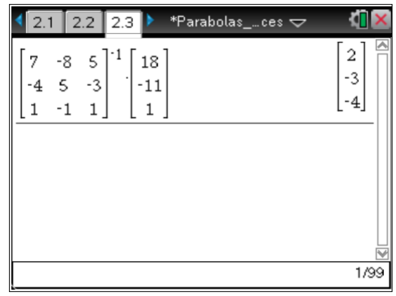
$3x - 5y + z = 14$ $3(2) - 5(-1) + 3 = 6 + 5 + 3 = 14$

$2x - 2y - z = 3$ $2(2) - 2(-1) - 3 = 4 + 2 - 3 = 3$

Teacher Tip: Be sure that students check their solutions. It is here that any errors in translating the system to a matrix equation will be revealed.

TI-Nspire Navigator Opportunity: Quick Poll
See Note at the end of the activity.

5. Solve the system below, using an inverse matrix. Use a Calculator application to perform your calculations.



- Press [ctrl] [doc▾] **> Add Calculator**.
- Select **MENU > Matrix & Vector > Create > Matrix**, and choose a matrix with 3 rows and 3 columns.
- Enter the data, and press OK or [enter]. (To move from one entry to the next, press [tab].)
- Press the right arrow (▶) to move outside the matrix.
- To obtain the inverse matrix, raise this matrix to the -1 power by pressing [^] [(–)] [1].
- Insert a multiplication symbol to the right of this inverse matrix.
- Add a new matrix with 3 rows and 1 column. Enter the data, and press [enter] to find the product.

$$7x - 8y + 5z = 18$$
$$-4x + 5y - 3z = -11$$
$$x - y + z = 1$$

a. In the spaces below, fill in the data that you entered into the matrices on your handheld.

<u>**Answer:**</u> $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 18 \\ -11 \\ 1 \end{bmatrix}$

b. Fill in your solution matrix in the spaces below.

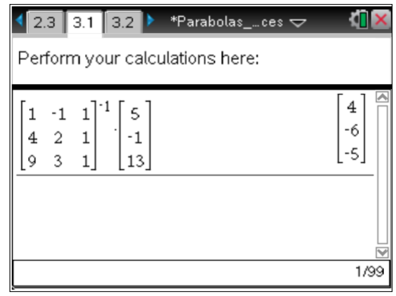
<u>**Answer:**</u> $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}$

> **Teacher Tip:** You might also want to provide students with several examples of systems with "missing terms" that will translate into 0's in a coefficient matrix.

TI-Nspire Navigator Opportunity: Quick Poll
See Note at the end of the activity.

Teacher Tip: The final problem asks students to find the equation of the parabola that passes through three specific points. Students are provided with steps that will guide them through using systems and matrices to solve the problem.

Move to page 3.1.



6. Explain how you could use an inverse matrix to find the equation of a parabola that passes through the points (–1, 5), (2, –1), and (3, 13).
   (Hint: Recall that quadratic equations are of the form $y = ax^2 + bx + c$. To write the equation of the parabola, you must use the three given points to set up a system of equations and solve for a, b, and c.)

   Sample Answer: Given the equation, $y = ax^2 + bx + c$, substitute the three given *x* and *y* values to obtain the equations:

   $$5 = a(-1)^2 + b(-1) + c \qquad\qquad 5 = a - b + c$$
   $$-1 = a(2)^2 + b(2) + c \quad \text{Simplifying, we get:} \quad -1 = 4a + 2b + c$$
   $$13 = a(3)^2 + b(3) + c \qquad\qquad 13 = 9a + 3b + c$$

7. Use the Calculator application on Page 3.1 to perform your calculations, and then fill in the information below:

Teacher Tip: You might want to perform the first substitution as an example for students. Substituting the first point (–1, 5) into the quadratic equation $y = ax^2 + bx + c$ yields $5 = a(-1)^2 + b(-1) + c$, which simplifies to $a - b + c = 5$. The other equations follow, and result in the system and matrix equation shown below.

- **System of equations:**

  Answer:
  $$5 = a - b + c$$
  $$-1 = 4a + 2b + c$$
  $$13 = 9a + 3b + c$$

- **Matrix equation:**

  Answer: $$\begin{bmatrix}1 & -1 & 1\\4 & 2 & 1\\9 & 3 & 1\end{bmatrix}\cdot\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}5\\-1\\13\end{bmatrix}$$

- **Solution:**

Answer:

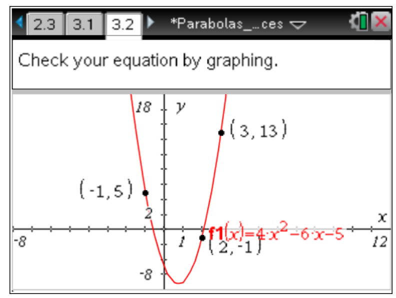
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 5 \\ -1 \\ 13 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -5 \end{bmatrix}$$

- **Equation of parabola:**

Answer: $y = 4x^2 - 6x - 5$

Teacher Tip: Students can confirm the solution (a = 4, b = –6, and c = –5) by substituting the coordinates and verifying that the equation holds.

Move to page 3.2.



8. On this page, press ctrl G to open the function entry line and enter the quadratic equation from Question 7. Check to make sure that the equation passes through all three points. Confirm the solution by substituting the values for *a, b*, and *c* into your system of equations and verifying the results.

Sample Answer: The graph of the parabola appears to pass through the three points, and that suggests we have found the solution. However, since the graph is not exact, and graphs sometimes lie, we also must check the solution analytically.

9. Use an inverse matrix to write the equation of the parabola that passes through the points (-1, 3), (1, –3), (2, 0). Write the equation below, and graph the parabola to check that it passes through the three points. Confirm the solution by substituting the values for *a, b,* and *c* into your system of equations and verifying the results.

Sample Answer:

We follow the same process as above. $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$

The equation for the parabola is $y = 2x^2 - 3x - 2$.

Teacher Tip: Students can go back to Page 3.1 or start a new calculator page to enter their matrices.

Tech Tip: To change the coordinates on the graph on Page 3.2, double-click on the *x*-coordinate of the point and change it to the new value. Double-click on the *y*-coordinate and change its value. Repeat this process for the other two given points.

TI-Nspire Navigator Opportunity: Quick Poll
See Note at the end of the activity.

Sample Answer:

Given the equation, $y = ax^2 + bx + c$, substitute the three given *x* and *y* values to obtain the equations:

$$3 = a(-1)^2 + b(-1) + c$$
$$-3 = a(1)^2 + b(1) + c$$
$$0 = a(2)^2 + b(2) + c$$

Simplifying, we get:

$$3 = a - b + c$$
$$-3 = a + b + c$$
$$0 = 4a + 2b + c$$

Our matrix equation is:

$$\begin{bmatrix} \boxed{1} & \boxed{-1} & \boxed{1} \\ \boxed{1} & \boxed{1} & \boxed{1} \\ \boxed{4} & \boxed{2} & \boxed{1} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \boxed{3} \\ \boxed{-3} \\ \boxed{0} \end{bmatrix}$$

After multiplying both sides of our equation by the inverse matrix, our solution is:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \boxed{1} & \boxed{-1} & \boxed{1} \\ \boxed{1} & \boxed{1} & \boxed{1} \\ \boxed{4} & \boxed{2} & \boxed{1} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boxed{3} \\ \boxed{-3} \\ \boxed{0} \end{bmatrix} = \begin{bmatrix} \boxed{2} \\ \boxed{-3} \\ \boxed{-2} \end{bmatrix}$$

Thus, the equation for the parabola is $y = 2x^2 - 3x - 2$.

Using the system of equations created above, we substitute $a = 2$, $b = -3$, and $c = -2$ as shown below.

$$3 = a - b + c \quad = \quad 2 - (-3) - 2$$
$$-3 = a + b + c \quad = \quad 2 - 3 - 2$$
$$0 = 4a + 2b + c \quad = \quad 4(2) + 2(-3) - 2$$

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to solve a system of simultaneous equations by using matrices.
- How to utilize matrices to write the equation for a parabola that passes through three given points.
- How to create and use their own matrices to solve problems.

Assessment

You might want to provide students with additional opportunities to solve systems of simultaneous equations.

TI-Nspire Navigator

Note

Name of Feature: Quick Poll

You might want to send a Quick Poll to have students submit their solutions.