Math Objectives

- Students will determine the product of two matrices.
- Students will calculate the inverse of the 2×2 matrix.
- Students will demonstrate that when [A][B] = [C], [A] = [C][B]$^{-1}$.

Vocabulary

- encryption key
- inverse matrix

About the Lesson

In this activity:

- A secret word, which is represented by a 2×2 matrix, has been coded using an encryption key.
- Students will be given the encryption key, along with the matrix that represents the coded word.
- By calculating the inverse of the encryption key, students will determine the secret word.
- Students will then create their own encryption key and secret words, exchange them with their partners, and decode them by calculating the inverse of their encryption keys.
- As a result, students will make conjectures about the relationships between matrices and their inverses.

A teacher .tns file has also been included with this activity which performs the required matrix multiplication and inverse steps as well.

TI-Nspire™ Navigator™ System

- Use Screen Capture to examine student progress.
- Use Live Presenter for student demonstrations.
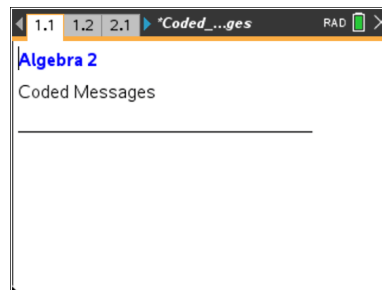
TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Change a math box
- Use a matrix template

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:
Student Activity
Coded_Messages_Student.pdf
Coded_Messages_Student.doc

TI-Nspire documents
Coded_Messages.tns
Coded_Messages_Teacher.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

> **Teacher Tip:** This activity is intended to be used as an application of inverse matrices to solve a system of equations. If this method has not yet been discussed in class, you may want to do so a few days before using this activity.
>
> Also, to keep the calculator file small and reusable for other types of coding, the students are instructed to go back and forth between the page that codes the message (page 1.2) and the page that decodes the message (page 2.1).

The secret word is represented by a 2 × 2 matrix, where each numeric element represents a letter. The secret word is encrypted by multiplying it by the encryption key, which contains random numeric elements that code the word. Your goal is to decode the secret word using the encryption key. This relationship can be modeled by the following equation:

$$\begin{bmatrix} \text{secret} \\ \text{word} \end{bmatrix} \begin{bmatrix} \text{encryption} \\ \text{key} \end{bmatrix} = \begin{bmatrix} \text{coded} \\ \text{word} \end{bmatrix}$$

You teacher's coded word is $\begin{bmatrix} -12 & 27 \\ -12 & 48 \end{bmatrix}$. Your teacher's encryption key is $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$.

1. The coded word can be decoded by isolating the secret word matrix. What inverse operation solves the equation for the secret word matrix?

    Answer: The secret word matrix is found by multiplying the inverse of the encryption key by the coded word matrix: . $\begin{bmatrix} \text{secret} \\ \text{word} \end{bmatrix} = \begin{bmatrix} \text{coded} \\ \text{word} \end{bmatrix} \begin{bmatrix} \text{encryption} \\ \text{key} \end{bmatrix}^{-1}$.

> **Teacher Tip:** This activity essentially requires students to solve the matrix equation [A] · [B] = [C] by multiplying both sides by the inverse of [B]. This is related to solving the general equation $a \cdot b = c$ for the variable a. In most cases, students would solve this equation by dividing both sides by the variable b. However, this equation can be solved by multiplying both sides by the multiplicative inverse of b, which is $\dfrac{1}{b}$ or b^{-1}.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 1 at the end of this lesson.

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2. Calculate the inverse of the encryption key.

Answer: $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{-1-2}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

> **Teacher Tip:** As an alternative to preforming the matrix multiplication by hand, consider having the students use the Scratchpad for their calculations.

3. Obtain your teacher's secret word matrix by multiplying the coded word matrix by the inverse of the encryption key. For example,

$$\begin{bmatrix} \text{secret} \\ \text{word} \end{bmatrix} = \begin{bmatrix} \text{coded} \\ \text{word} \end{bmatrix}\begin{bmatrix} \text{encryption} \\ \text{key} \end{bmatrix}^{-1}$$

Answer: $\begin{bmatrix} -12 & 27 \\ -12 & 48 \end{bmatrix}\left(-\frac{1}{3}\right)\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -12(1)+27(-1) & -12(-2)+27(-1) \\ -12(1)+48(-1) & -12(-2)+48(-1) \end{bmatrix}$

$$= -\frac{1}{3}\begin{bmatrix} -39 & -3 \\ -60 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 1 \\ 20 & 8 \end{bmatrix}$$

TI-Nspire Navigator Opportunity: *Live Presenter*
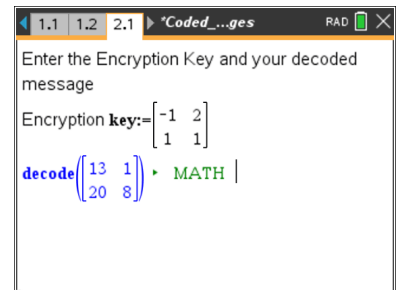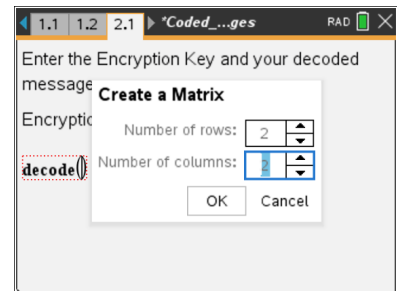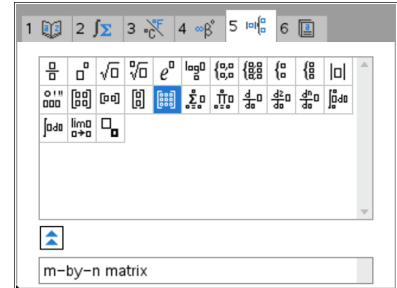**See Note 3 at the end of this lesson.**

> **Teacher Tip:** Many students make mistakes with this method due to multiplying the matrices in the wrong order. It will be beneficial to lay out the following steps: Since the system you are trying to solve is
> **[secret word][encryption key] = [coded word]**, it follows that…
> **[secret word][encryption key][encryption key]$^{-1}$ =**
> **[coded word][encryption key]$^{-1}$**

Move to page 2.1.

4. Turn your teacher's secret word matrix into an actual word using the **decode( )** command. Click inside the parentheses and insert a 2 × 2 matrix template by selecting the catalog and selecting the 2 × 2 matrix template. Enter the secret word *matrix* and press enter. Write your teacher's word below. If the word is unreadable, recheck your calculations in questions 2 and 3.

Answer: Math

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response), Live Presenter*
See Note 4 at the end of this lesson.

Move to page 1.2.

5. It is now time for you to create your own secret message. First, double-click the "encryption key" matrix and change the current values to four random integers. Then choose a secret word that contains four letters.Click on the empty box in the **code( )** command, type your word, and press (enter). The resulting matrix is your coded word matrix. Record your encryption key and coded word matrix below.

    a. Your encryption key: Your secret word matrix:

    Sample answer: $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 8 & 5 \\ 12 & 16 \end{bmatrix}$

    b. Calculate your coded word matrix by multiplying your encryption key by your secret word matrix.

$$\begin{bmatrix} \text{secret} \\ \text{word} \end{bmatrix} \begin{bmatrix} \text{encryption} \\ \text{key} \end{bmatrix} = \begin{bmatrix} \text{coded} \\ \text{word} \end{bmatrix}$$

Sample answer: $\begin{bmatrix} 8 & 5 \\ 12 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 18 \\ -4 & 44 \end{bmatrix}$

Move to page 2.1.

6. Give your encryption key and coded matrix to a neighbor to decode. Your neighbor will give you an encryption key and a coded word matrix, which you can record below. Your goal is to determine your neighbor's secret word.

   a. Your neighbor's encryption key: Your neighbor's coded word matrix:

   Sample answer: $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 18 \\ -4 & 44 \end{bmatrix}$

   b. Calculate the inverse of your neighbor's encryption key.

   Sample answer: $\dfrac{1}{3} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

   c. Calculate your neighbor's secret word matrix by multiplying the coded word matrix by the inverse of the encryption key. For example,

   $$\begin{bmatrix} \text{secret} \\ \text{word} \end{bmatrix} = \begin{bmatrix} \text{coded} \\ \text{word} \end{bmatrix} \begin{bmatrix} \text{encryption} \\ \text{key} \end{bmatrix}^{-1}.$$

   Sample answer: $\begin{bmatrix} 8 & 5 \\ 12 & 16 \end{bmatrix} = \dfrac{1}{3} \begin{bmatrix} 3 & 18 \\ -4 & 44 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

   d. Turn your neighbor's secret word matrix into an actual word using the **decode( )** command. First, change the encryption key by double-clicking the matrix. Enter your neighbor's encryption key, which you recorded in question 6a. Then enter the secret word matrix into the **decode( )** command. Click inside the parentheses and insert a 2×2 matrix by pressing ⌨ and selecting the 2×2 matrix template. Enter the secret word matrix and press (enter). Write your neighbor's word below.

   Sample answer: Help

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*

See Note 5 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students know:

*   When [A][B] = [C], [A] = [C][B]$^{-1}$.

Assessment

1. In the matrix equation $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 18 \\ 7 & 12 \end{bmatrix}$, determine the values for $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$.

    Answer: $\begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix}$

2. Decode $\begin{bmatrix} 7 & 23 \\ -8 & 18 \end{bmatrix}$ if the encryption key was $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

    Answer: Home

Consider using the TI-Nspire Navigator to have students submit their answers using a Quick Poll (Open Response).

TI-Nspire Navigator

Note 1
Question 1, *Quick Poll (Open Response):* Ask students to submit their answer to question 1.

Note 2
Question 2, *Live Presenter:* To help students verify their calculations with the handheld, select a student presenter and ask this student to open the Scratchpad. Type $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}^{-1}$ and press $\boxed{\text{enter}}$.

Note 3
Question 3, *Live Presenter:* To help students verify their calculations with the handheld, select a student presenter and ask him to open the Scratchpad. Guide the student through the matrix multiplication on the handheld.

Note 4
Question 4, *Live Presenter, Quick Poll (Open Response)*: Consider using Live Presenter to demonstrate how to use the matrix template as well as how to change values of the encryption key. Use the Quick Poll to have students submit their answer to question 4.

Note 5
Question 6, *Quick Poll (Open Response):* Ask students to submit the secret words their neighbors gave to them.