Radical Functions
ID: 8977

Activity Overview

In this activity, students use a nomograph to investigate functions defined by square roots. Nomographs consist of two or more parallel axes, one for inputs and another for outputs. Input, output pairs that belong to the function are graphed as corresponding points on the axes connected by a ray drawn from the input to the output.

Topic: Rational and Radical Functions

* *Evaluate a radical function for any real value of its variable.*

* *Graph radical functions and inequalities.*

Teacher Preparation and Notes

* *This investigation could be used as a first introduction to functions defined using radicals. Students should be able to evaluate a function for a given input, and should understand square roots. They are led to discover the domain of a simple radical function (problem 1) and a more complex one (problem 3) that involves a quadratic under the square root*

* *This activity is designed to be used in Algebra 2 or Precalculus as an introduction to functions defined using radicals.*

* ***To download the student worksheet and calculator program NOMORAD, go to education.ti.com/exchange and enter "8977" in the keyword search box.***

Associated Materials
* *RadicalFunctions_Student.doc*
* *NOMORAD.8xp*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

* *Rational Functions (TI-84 Plus family) — 8967*

* *Exploring the Exponential Functions (TI-84 Plus family) — 11978*

A **nomograph** is similar to a function machine in that it relates a number from one set (the domain) to a number in a second set (the range). Each set of numbers is represented in a pair of vertical number lines; the domain is on the left, and the range is on the right. According to the function rule, an element of the domain is mapped to its corresponding range element, and this mapping is depicted by an arrow.

Prior to beginning Problem 1, review domain and range, and ensure that students understand how to use the model.

Problem 1 – Domain of a Simple Radical Function

In this problem, students get acquainted with the idea of a nomograph by investigating a simple radical function.

Students observe the mapping as they enter different values for *x* or adjust the position of the input value using the up and down arrow keys. Pressing **9** exits the nomograph and returns to the menu.
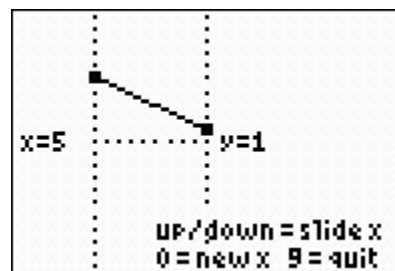
At a certain point, students will see the mapping vanish. This is likely to surprise, but illustrates the idea of the domain of a function quite nicely.

Questions that follow will lead students to state the domain of the function.
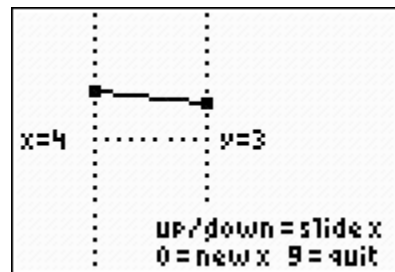
Student Worksheet Solutions

1.

Input		Output
4	$\rightarrow$	0
6	$\rightarrow$	1.414
8	$\rightarrow$	2
13	$\rightarrow$	3
20	$\rightarrow$	4
29	$\rightarrow$	5

2. near $x = 4$

3. The function is undefined when $x < 4$. When point *x* is dragged to a value less than 4, there is no output from the function, so the arrow that shows the mapping from input to output disappears.

4. $\{x \mid x \geq 4\}$

Problem 2 – Guess the Radical Function

This nomograph represents an unknown function. By repeatedly entering different inputs, the student should be able to discover the function's rule.

When students have conjectured a rule, they should record it on their worksheets and check it. The rule is checked by selecting an input number, applying the rule, and predicting the output number.

Student Worksheet Solutions

5. $x = -5.1$

6.

Input		Output
–4	→	1
–1	→	2
4	→	3

7. $f(x) = \sqrt{x+5}$

8. $\{x \mid x \geq -5\}$

9. a. $g(x) = \sqrt{x-10}$

**b.** $h(x) = \sqrt{x-13}$

c. $j(x) = \sqrt{29-x}$

d. $k(x) = \dfrac{1}{\sqrt{10-x}}$

Problem 3 – A More Complicated Radical Function

In this problem, students investigate a function with a quadratic expression under the square root, $f(x) = \sqrt{(x-1)(x+4)}$. Since the quadratic changes sign twice, there are <u>two</u> points at which the ray showing the mapping from inputs to outputs disappears. As before, students determine the domain of the function after evaluating the function and observing the mapping.

Student Worksheet Solutions

10.
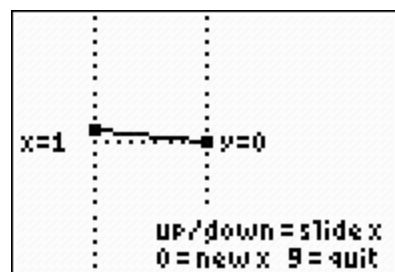
| Input		Output
1	→	0
–4	→	0
5	→	6
–8	→	6
–2	→	does not exist

11. $x = 1$ and $x = -4$

12. a. yes

b. 0 for both

13. $\{x \mid x \geq 1 \text{ or } x \leq -4\}$

Extension - Connection to Cartesian Graphs

Explain to students that as they investigate the nomographs, the calculator records the inputs and outputs. Now they can take a look at this data. Students should first choose the **ClearData** option from the main menu, which will clear out the data collected during their previous work. Then they should choose one of the rational functions and enter 10–15 different input values.
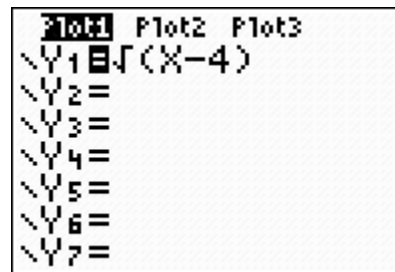
If using Mathprint OS:

The square root symbol will go all the way across the expression. When students press 2nd [√] the flashing cursor will be under the radicand. To move out of the radicand, press ▶.

Note: Parentheses are not needed under the radicand.
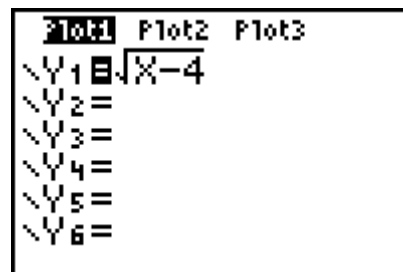
Students will next graph the data they collected as a scatter plot. They should see that the only points on the graph are ones with an *x*-coordinate greater than or equal to 4 and a *y*-coordinate greater than or equal to 0. Next have students enter the equation $f(x) = \sqrt{x-4}$ in **Y₁** and graph it. The graph should go through all the points in the scatter plot.

Student Worksheet Solutions

14. The domain of this function includes only values of *x* that are greater than or equal to 4.

15. The square root of a number is always greater than or equal to 0. The outputs from this function are the *y*-coordinates on its Cartesian graph.

16. The graph goes through all the points on the scatter plot.