Binomial Probabilities

About the Lesson

In this activity, students begin with a hands-on experiment of rolling a die and keeping track of the numbers of successes and failures. They then simulate their experiment by using the **randBin** command. Next, they use the **binomPdf** command to find the theoretical probabilities and compare their experimental probabilities to the theoretical probabilities. Students also use the **binomCdf** command to find cumulative probabilities. The activity concludes with deriving the formula for finding binomial probabilities. As a result, students will:

- Calculate the probability of *r* successes in *n* Bernoulli trials for a particular experiment.
- Use the binomial probability density function to verify the probabilities calculated for n Bernoulli trials.

Vocabulary

- experimental probability
- theoretical probability
- probability density function

Teacher Preparation and Notes

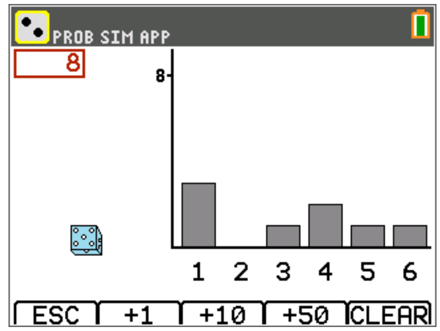
- This activity is designed for use in an Algebra 2 classroom. It can also be used in an introductory Statistics class.
- You may use standard six-sided dice for students to roll. If dice are not available, you can use spinners divided into six equal sections.
- This activity assumes basic knowledge of probability, including the difference between experimental and theoretical probabilities, as well as arrangements and combinations. Students should also know what is meant by the complement of an event.
- The first part of the activity uses the **Prob Sim** App to roll a die. This experiment can also be performed using a real die.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Binomial_ Probabilities_Student.pdf
- Binomial_ Probabilities _Student.doc

Problem 1 – Experimental Probability

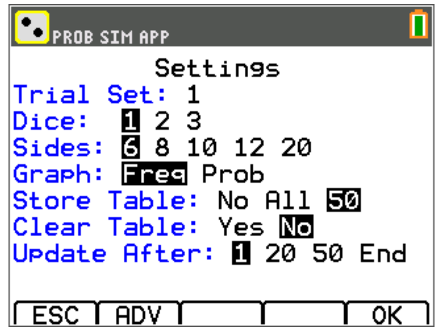
Introduce or review the following rules for a binomial experiment:

- There are n independent trials, or observations.
- There are two possible outcomes, or categories: a success or a failure.
- The probability of a success, p, remains constant throughout the experiment. The probability of a failure, q, is also constant; $q = 1 – p$.

To do the experiment, students have two options, (1) use the **Prob Sim** app or (2) use a real die. Both options are explained below.

Option 1:

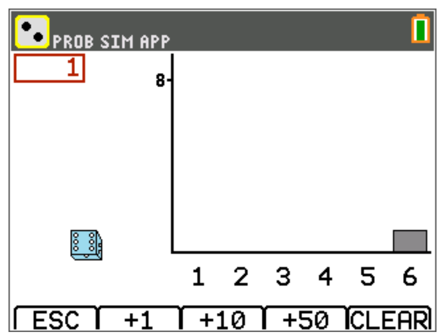
Press apps and select **Prob Sim**. Choose the Roll Dice simulation.
Press zoom to select the **SET** menu and change the settings to those shown at the right. Then press graph to select **OK**.



Press window to select **ROLL**. This will roll the die once. Perform the experiment of rolling the die five times and recording the result where a 6 is a success and any other number is a failure.

Press graph to clear the experiment.

Students are to repeat this experiment nine more times, using **Table 1** on their worksheet to record and keep track of their results.



Option 2:

Give each student a single die. Each student should perform the experiment of rolling the die five times and recording the result where a 6 is a success and any other number is a failure.

Have students repeat this experiment nine more times, using **Table 1** on their worksheets as a place to record and keep track of their results. A sample table is shown at right.

Next, students can complete **Table 2** by recording the number of successes in each of the 10 experiments. In the first row, they should write the number of successes, and in the second row, they should record the percent of successes. The numbers should sum to 10, the percents should sum to 1.

<table>
<tr><th>Successes</th><th>Failures</th></tr>
<tr><td></td><td>卌</td></tr>
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A sample table is shown below. (It corresponds to the sample tally table on the previous page.) The last row shows experimental binomial probabilities. Ask various students what their experimental probability was for exactly two successes.

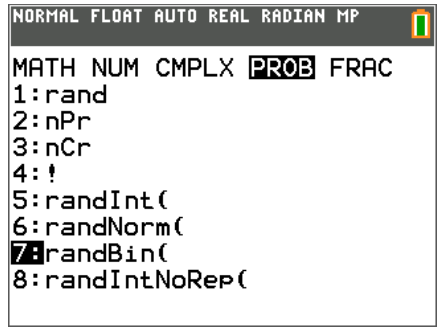
	0	**1**	**2**	**3**	**4**	**5**
Number of Successes	5	3	2	0	0	0
Percent of Successes	0.5	0.3	0.2	0	0	0

Explain that, in this scenario, $n = 5$ because there were five trials per experiment. The experiment was performed 10 times. Ask students how they think performing the experiment 100 times would affect the probability distribution (the values in the last row of the table). They should predict that the results would become closer to the theoretical probability. Ask students if they would want to sit there and repeat the experiment 100 or more times.

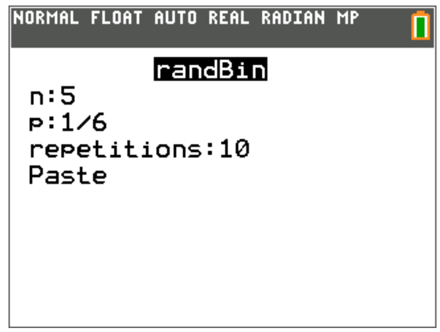
Explain to students that their graphing calculator has a function that will simulate binomial experiments. They will learn to use it and see how the simulated results compare to their actual results.

If using the **Prob Sim** app, have students exit the app.

To simulate the same experiments performed earlier, students are directed on the worksheet to use the **randBin(** command. To access the command they can press [math], arrow over to **PROB**, and choose it from the list.



Then they need to enter the required arguments as shown to the right. The **n** value refers to the number of trials per experiment, **p** refers to the probability of success in each trial, and **repetitions** represents the number of experiments.



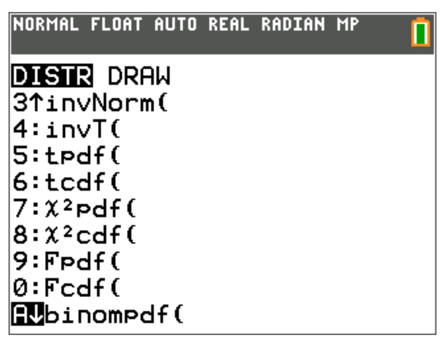
Pressing [enter] reveals a list with the number of successes per experiment appears. Use the right arrow to see all of the numbers.

Teacher Tip: This would be a good opportunity to ask students whether or not their list matches the one in the screenshot. Ask them why or why not.

They should use this information to complete **Table 3** on their worksheet and then compare these simulate results to their Table 2 experimental results.

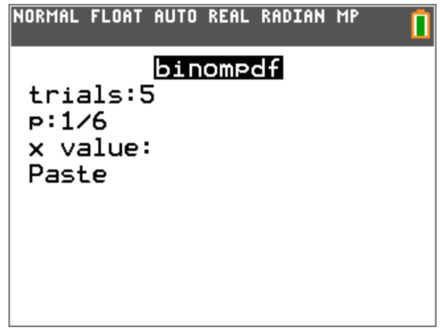
Problem 2 – Theoretical Probability

The probabilities in Tables 1 and 2 are experimental. For students to find the theoretical probabilities, they will use the **binomPdf** (binomial probability density function) command. This command is found in the **DISTR** menu, which is accessed by pressing [2nd] [vars][distr].



Students are to select the option **A:binomPdf(** and enter **5** as the number of **trials** and **1/6** as the value of **p.** Choose **Paste** and press [enter]. The list of theoretical probabilities appears, beginning with *P*(0) and ending with *P*(5). Students can arrow to the right to view all the values.

Note: Students will leave the **x value:** field blank.

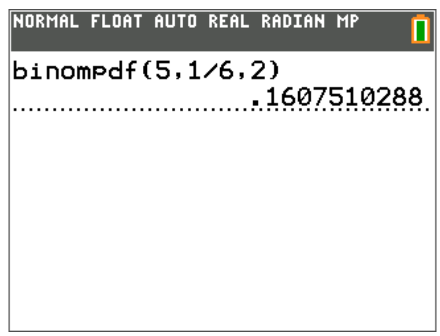


Have students enter the theoretical probabilities into **Table 4** on their worksheet. Next, they should compare these theoretical probabilities to their experimental probabilities, both performed and simulated.

> **Teacher Tip:** Ask questions about the probability distribution such as: *Why do the probabilities get closer to zero towards the right side of the table?*

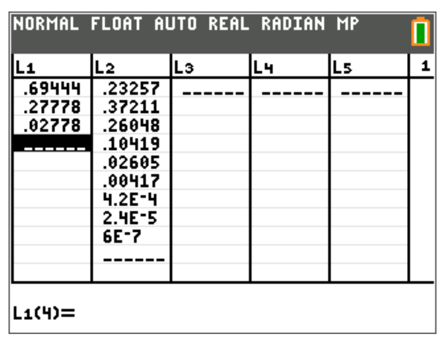
Explain to students that, instead of seeing an entire probability distribution as they had just completed, they can choose to see the probability of any given number of successes by adding a third argument to the **binomPdf** command.

Students can calculate the probability of exactly two successes by returning to the **binomPdf** function and enter a **2** for the **x value**.



Instruct students to find **binomPdf(2,1/6)** and **binomPdf(8,1/6)**. Ask how and why the distributions differ from **binomPdf(5,1/6)**. Also ask which gives the greater probability of exactly two successes and why.

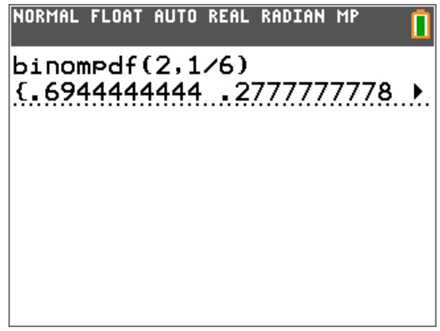
To view the probabilities side-by-side, have students store **binomPdf(2,1/6)** in list L1 and **binomPdf(8,1/6)** in list **L2**.



1. Compare the experimental probabilities to the theoretical probabilities.

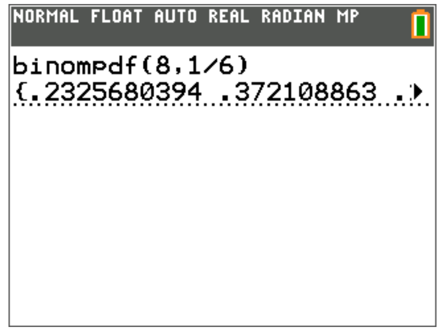
Sample Answer: The theoretical probabilities offer possibilities for all outcomes where the experimental probabilities only offer possibilities for a few of the outcomes.

2. Find **binomPdf(2,1/6)** and **binomPdf(8,1/6)**.



Answers:

binomPdf(2,1/6) =
$P(0) \approx 0.6944$, $P(1) \approx 0.2778$, $P(2) \approx 0.0278$



binomPdf(8,1/6) =

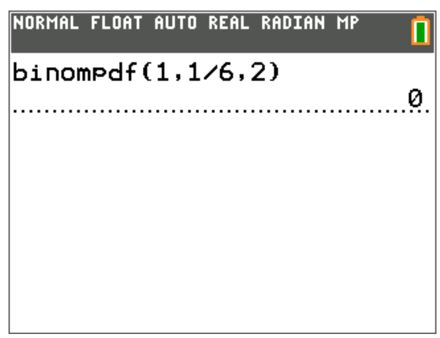
$P(0) \approx 0.2326$, $P(1) \approx 0.3721$, $P(2) \approx 0.2605$,
$P(3) \approx 0.1042$, $P(4) \approx 0.0261$, $P(5) \approx 0.0042$,
$P(6) \approx 0.0004$, $P(7) \approx 0.00002$,
$P(8) \approx 0.0000006$

Students may calculate the above two probability density functions as shown to the right or they refer back to their table.

3. Explain how and why the probability distribution changes. Which gives a greater probability of exactly 2 successes? Why?

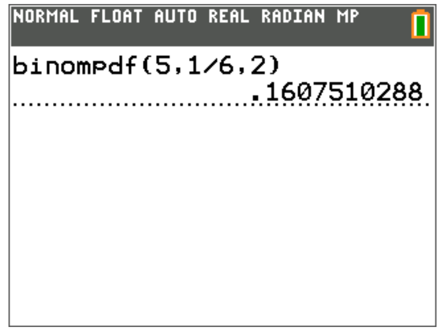
Answer: The binomPdf(8,1/6) gives a greater probability of exactly 2 successes. The greater the number of trials, the more likely there will be exactly 2 successes.

4. Find **binomPdf(1,1/6,2)**. Explain why you get this result.



Answer: 0; It is impossible to obtain two successes in only one trial.

Next, students will use the **binomCdf** command, which gives cumulative probabilities. It is also found in the **DISTR** menu ([2nd] [vars][distr]).



5. Use **binomCdf(5,1/6,2)** to find the probability of two or fewer successes.

Answer: ≈ 0.1608

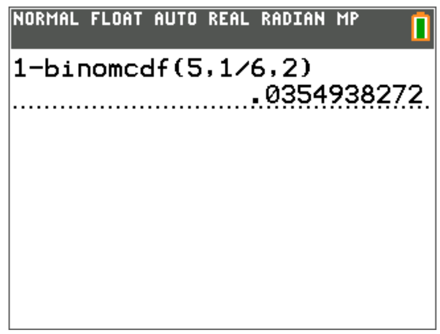
Note: This result is consistent with $P(0) + P(1) + P(2)$.

Allow students to check other cumulative probabilities as well, including **binomCdf(5,1/6,4)** and **binomCdf(5,1/6,6)**.

6. Then find the probability of at least three successes.

Answer: ≈ 0.0355.

Note: This can easily be found by calculating the complement of **binomCdf(5,1/6,2)**, as shown to the right. This is equivalent to $P(3) + P(4) + P(5)$.



Problem 3 – Using the Formula

7. Below, list all the arrangements of two successes and three failures in five trials. One arrangement is done for you.

SSFFF

Answer: SSFFF, SFSFF, SFFSF, SFFFS, FSSFF, FSFSF, FSFFS, FFSSF, FFSFS, FFFSS

8. How many arrangements are there?

Answer: There are 10 arrangements.

9. What is the probability of each arrangement? Why?

Answer: Because the trials are independent, we must multiply the probabilities of successes and failures. For example, for the arrangement SSFFF, the probability is $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{125}{7776}\approx 0.0161$.

Although the factors are in a different order, the probability of each arrangement is the same.

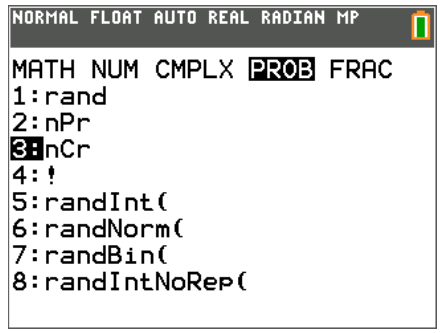
10. What is the total probability of two successes in five trials?

Answer: Because there are 10 arrangements, the total probability of two successes in five trials when $p = \frac{1}{6}$ is $10\left(\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6}\right) = \frac{1250}{7776} \approx 0.1608$.

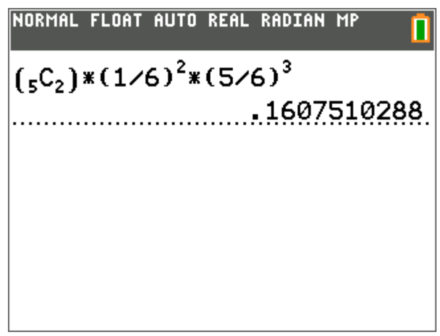
11. What is the formula for finding a binomial probability?

Answer: The binomial probability of r successes in n trials is $P(r) = {}_nC_r \cdot p^r \cdot q^{n-r}$, where p is the probability of success and q is the probability of failure.

Access the **Combinations** command by pressing [math], arrow over to the **PROB** menu, and choose it from the list by pressing [enter]. Type the value of n before the command and the value of r after it.

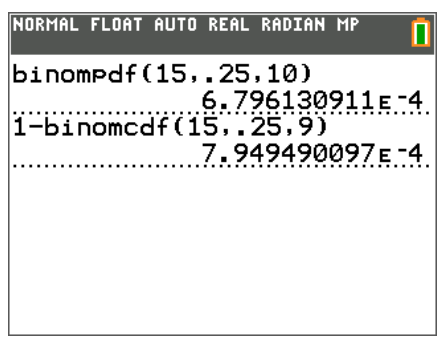


Note: Using the formula $P(r) = {}_nC_r \cdot p^r \cdot q^{n-r}$ yields the same answer as the **binomPdf(5,1/6,2)** command.



Allow students to work independently to answer Question 12 on the worksheet.

12. The probability of randomly guessing any correct answer on a multiple-choice test is 0.25. The test has 15 questions. Find the probability of guessing:



a. Exactly 10 answers correctly

Answer: $P(10) \approx 0.00068$ or 0.068%

b. At least 10 answers correctly

Answer: $P(\text{at least } 10) \approx 0.000795$ or 0.0795%