Pythagoras Squared

Teacher Notes and Answers

7 **8** **9** 10 11 12

TI-Nspire | Investigation | Student | 60 min

Teacher Notes: This activity uses the Geometry application to provide a relatively simple 'proof' of Pythagoras's theorem. There are some underlying assumptions regarding triangle congruency which may be explored in further depth.

Squares and Triangles

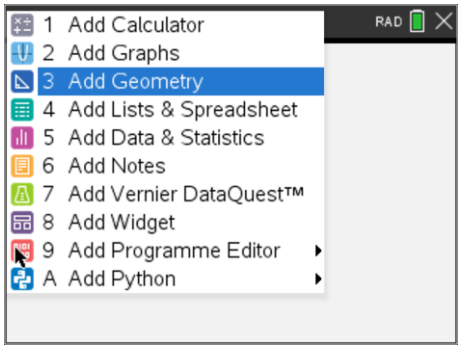
Start a new document and insert a Geometry Application.

There are two ways to create a new document:

- Home screen: [on]
- Keyboard shortcut: [ctrl] + [N]

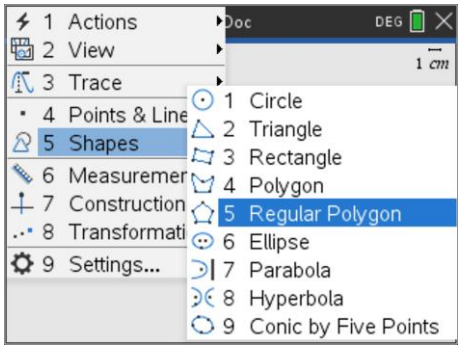
You may be prompted to save the existing document, select your preferred option (yes/no) accordingly.

When the new document is displayed, select option **3: Geometry**



The next step is to insert a square, this will be done using the shapes menu and selecting regular polygon.

[menu] > **Shapes** > **Regular Polygon**

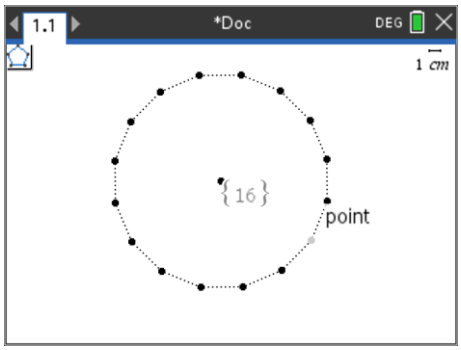


When the drawing tool is in use the mouse changes to a pencil.

Click in the middle of the screen to place the 'centre' of the polygon.

Slide your finger across the track-pad, 16 connected vertices will appear.

Click again to create the first vertex of your polygon.



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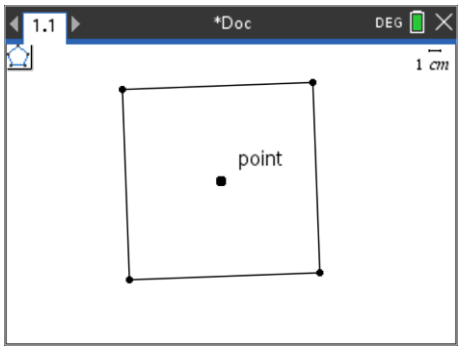
Moving the pointer in a clockwise direction from your vertex will decreased the number of vertices, and therefore sides, on your polygon.

The aim is to create a square, click the mouse again when there are four vertices (and four sides) for your polygon.

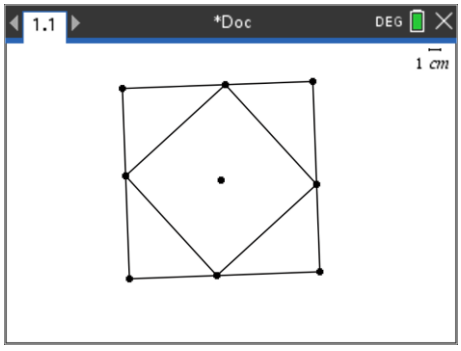


The regular polygon tool is still active. Another square needs to be created with the same centre and the vertices on the sides of the original square. (Shown opposite)

The tips tool will prompt when the centre, and vertex are 'on' the previous centre and side.

Once you have finished, press: esc to release the polygon tool.

Note: If you make a mistake, press either esc or ctrl + Z to undo the mistake.

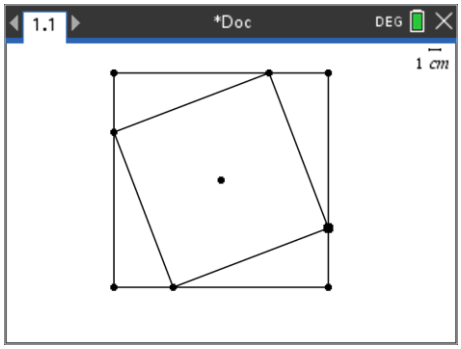


Testing

To test your construction, place the mouse over one of the vertices on the inner square, then click (⬚) and hold for approximate one second or press: ctrl followed by click to grab the vertex.

You can now move the mouse and watch the inner square move.

To release the grip on the vertex, either click or press esc.



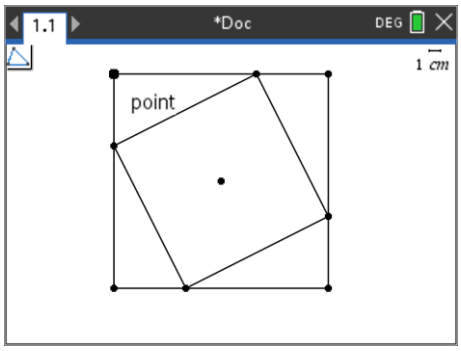
The regions between the outer and inner squares forms four triangles, we can see them, but the digital world is unaware of their existence. The next set of instructions requires you to draw, colour and measure the four triangles. It is important to ensure the vertices of the triangles land precisely so they move with the square as it is rotated.

Draw the first triangle:

menu > **Shapes** > **Triangle**

The triangle drawing tool icon appears in the top left corner of the screen. Place the mouse over one of the outer square's vertices. The tool prompt says "point", this means the triangle's vertex will be placed on the square's vertex. Click to place the vertex then proceed to the next two vertices accordingly to form the triangle.

Repeat this process for the three remaining triangles, press esc when you have finished creating the triangles.



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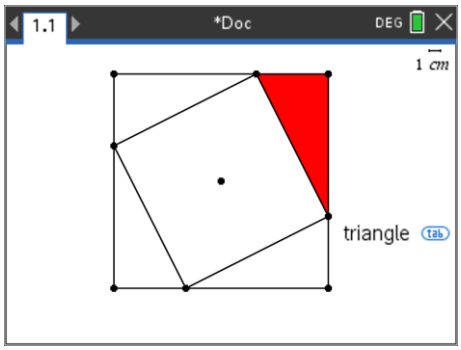
The next step is to colour the triangles. Move the mouse over the top of one of the triangles (one of the sides). The text prompt "triangle" appears, press:

ctrl menu > Colour > Fill Colour (select a colour)

This key combination is equivalent to a 'right mouse click' on a computer. Select colour, fill colour then select a colour.

Repeat this process for the remaining three triangles.



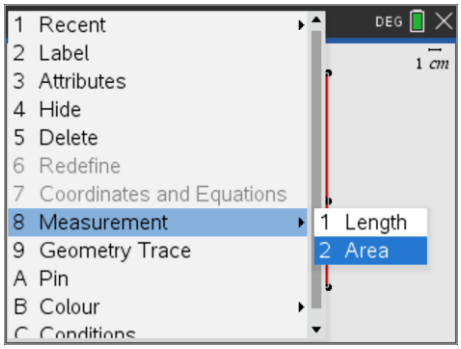
The next step is to measure the area of the squares and the triangles.

Place the mouse over outer square, the prompt will say "triangle" because the triangles are sitting on top of the original square. An additional prompt "tab" appears, press:

tab (The prompt will change to 'square')

ctrl menu > **Measurement > Area**

The area of the outer (original) square will be displayed. Repeat this process to measure the area of the inner square and the area of one of the triangles.



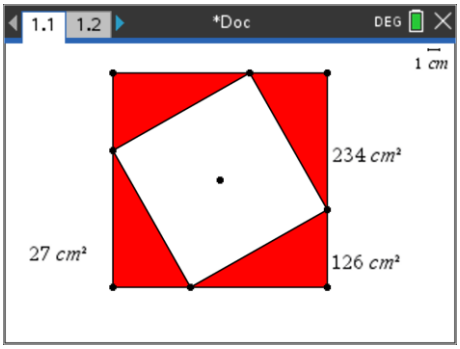
Question: 1.

Write down your three area measurements: Large or Outer Square (L), Triangle (T) and Small or Inner Square (S).

a) Write down and perform a calculation that relates the three measurements.

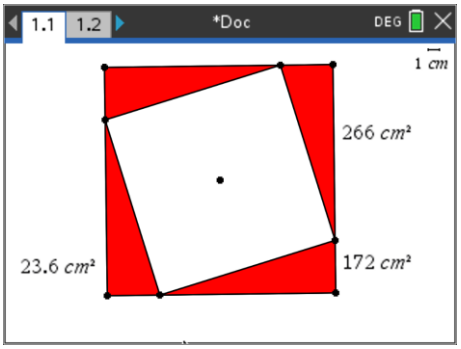
Answer: L – 4T = S [Students subtract the total area of the four triangles from the area of the large or outer square to obtain the area of the inside or smaller square.
Example: 234 – 4 x 27 = 126

Note: The example shown here involves a specific case where the measurements all appear as integers. The display has been set to 3 digits, however, if the inner square is rotated slightly, the triangle areas will include a decimal place and introduce some 'rounding' issues. These should be discussed with students.



a) Rotate the inner square a little more, record the new measurements and calculations.

Answer: Sample shown ... in this case the size of the larger square has also been changed.
Example: 266 – 4 x 23.6 ≈ 172 (171.6)



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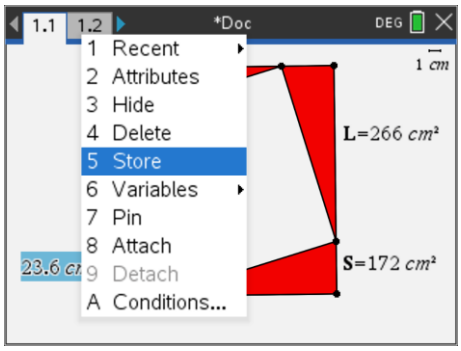
The measurements captured in the Geometry application can be stored and recalled elsewhere in the same problem.



Move the mouse over the measurement for the area of the large square:

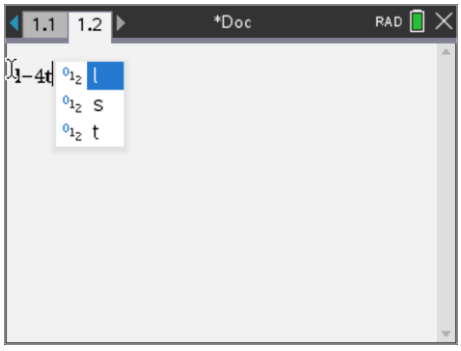
ctrl menu > **Store**

Store the measurement for the large square in "L".

Repeat this process for the area of the smaller square: "S" and the are of the triangle: "T".

Insert a calculator application:

ctrl I > **Add Calculator**

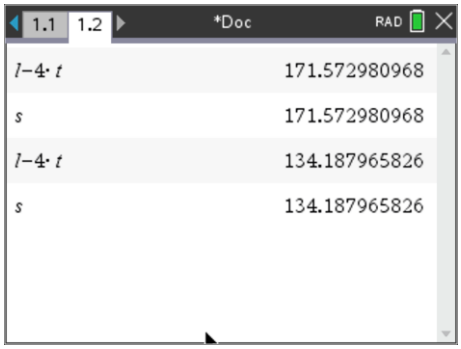


Variables stored in the Geometry application can be retrieved directly using the keyboard or by pressing the variable key: var

Type the expression:

L – 4T (variables will appear in lower case and bold)

Compare the result with the value stored in S. (Smaller square)



You may have noticed that there are now two pages in your current document, page 1.1 and 1.2. We need to get back to page 1 (1.1).

ctrl (Left side of navigation pad)

Rotate the inner square, return to the calculator page (1.2)

ctrl (Right side of navigation pad)

Copy and paste your previous calculations to see the new results.

Note: To copy and paste, you can use Ctrl + C and Ctrl + V, alternatively, simply arrow up to the expression (highlights the expression) then press enter

Question: 2.

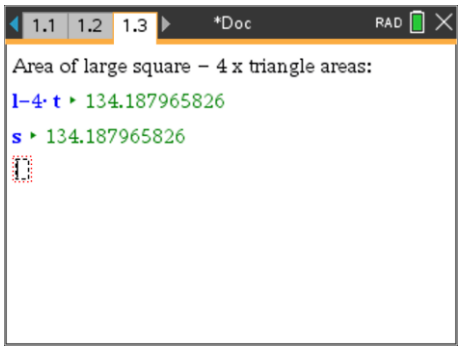
Comment on the calculations in the calculator application:

Answers: Possible comments include:

- More *accuracy* or more *decimal places* are displayed.
- Calculations are static (not updated automatically)
- The results are now exactly the same. (Need to be careful with these comments!)

The Notes application is also capable of performing calculations. Insert a Notes application:

ctrl I > **Add Notes**



Notes can contain a mixture of text and calculations. Include some text to describe what you are calculating, then press:

ctrl M

This inserts a 'Maths Box'. Enter your expression from the previous page into the maths box.

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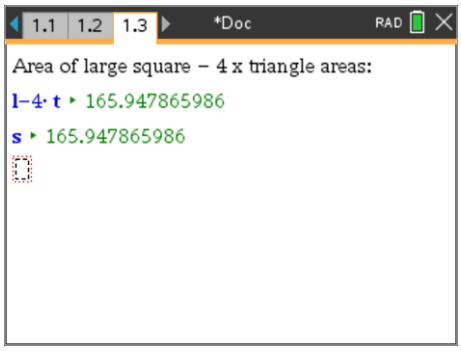
Navigate back to the Geometry application, rotate the inner square, then return to the Notes application once again.

Note: You can use ctrl (Top of Navigation pad).

This option presents all the applications in a 'slide' view. You can navigate to the page you want and press: enter



Question: 3.

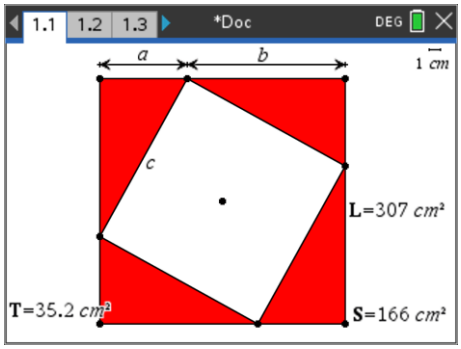
Comment on the calculations in the Notes application:

Answers: Possible comments include:

- Calculations are automatically updated.
- Notes (text) can make the calculations more meaningful.

Generalising the Solution

Some labels have been added to the diagram. Use these labels to help answer the following questions.



Question: 4.

Area formulas:

a) Write an expression for the area of the smaller (inside) square.

Answer: c^2.

b) Write an expression for the side length of the larger square and hence an expression for the area of the larger square.

Answer: Side length = $a + b$. $\therefore$ Area = $(a + b)^2$.

c) Write an expression for the area of one of the red triangles.

Answer: $\frac{1}{2} \times base \times height = \frac{1}{2} \times a \times b$

d) In the calculator application, write an equation relating the area of the large square, triangles and small square in terms of a, b and c.

Answer: $(a+b)^2 - 4 \times \left(\frac{1}{2}ab\right) = c^2$ [Calculator does the simplification, need 'x' between a and b]

$a^2 + b^2 = c^2$

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