Pythagoras 3D

TI-Nspire™   Investigation   Student   60 min

Introduction

You should already be familiar with using Pythagoras' Theorem to calculate the length of an unknown side of a right triangle. Did you know that you can extend Pythagoras' Theorem for use in three dimensions? This means you can determine the length of the diagonal of a rectangular solid. In this activity, you will have the opportunity to explore different ways to use TI-Nspire technology to calculate Pythagoras' Theorem in three dimensions.
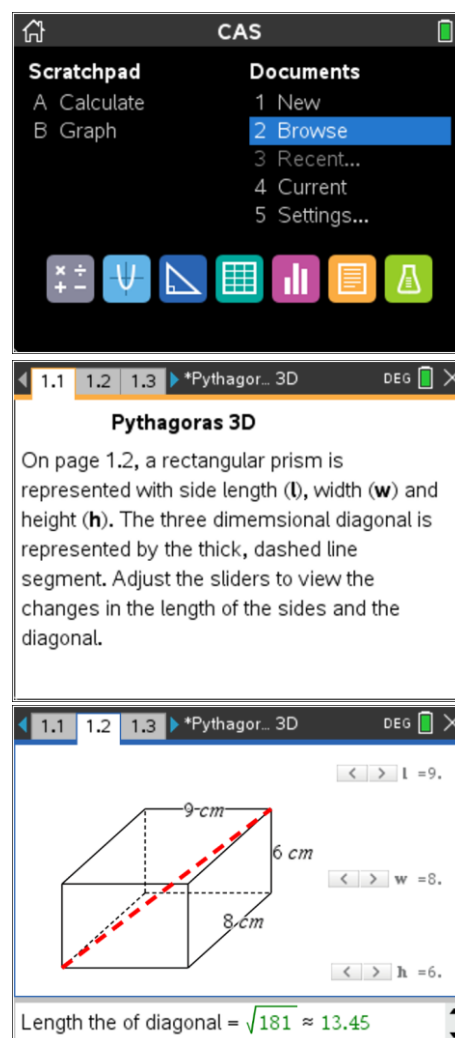
Part A: Using the Calculator Application

This activity requires access to the "Pythagoras 3D" TI-Nspire document. This document should be loaded on your device before proceeding.

Once the document is on your handheld, press [**home**] and select **Browse**. Locate the "Pythagoras 3D" document and press [**enter**] to open.
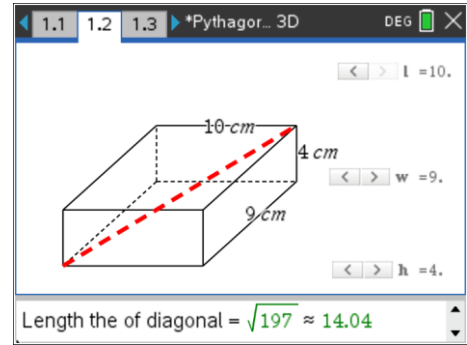

Page 1.1 of the activity gives an overview.

Navigate to page 1.2 and adjust the sliders to change the values of the length (l), width (w) and height (h). The exact and approximate values of the diagonal will appear in the small window below the shape.

Author: Neale Woods

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Observe how the length of the diagonal changes as the dimensions change.

Page 1.4 is a Calculator application showing the calculation to find the length of one diagonal.

To find the approximate answer, press **[ctrl] + [enter]** or include a decimal point with at least one of the values.

For repeated calculations, use the automatic copy and paste feature.

Press the **up** arrow to highlight a previous entry then press **[enter]** to copy it to the edit line. Then, replace the values.
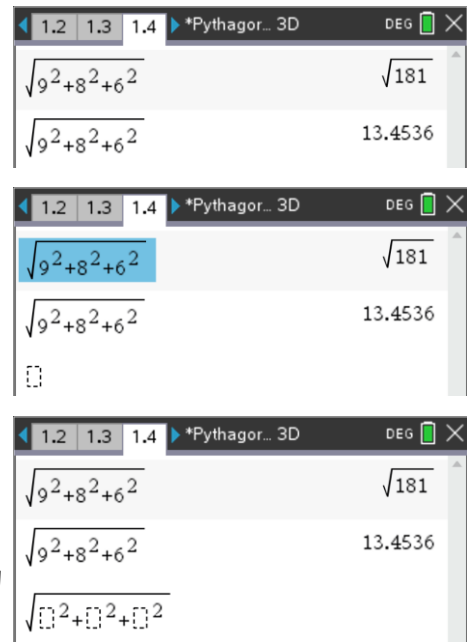
Question: 1
Complete the table below by selecting four different dimensions from page 1.2. Compare them to your own calculations using the Calculator on page 1.4 (see example below).

Dimensions			Page 1.2		Calculator page	
length (l)	width (w)	height (h)	Exact	Approximate	Exact	Approximate
10	9	4	$\sqrt{197}$	14.04	$\sqrt{197}$	14.0357

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Part B: Using the Lists & Spreadsheet Application

We can also use a Lists& Spreadsheet page to calculate values.

On page 1.6, the first three columns have the headings **length**, **width** and **height**. The fourth column has the heading **diagonal** and uses a formula to calculate the length of the diagonal.
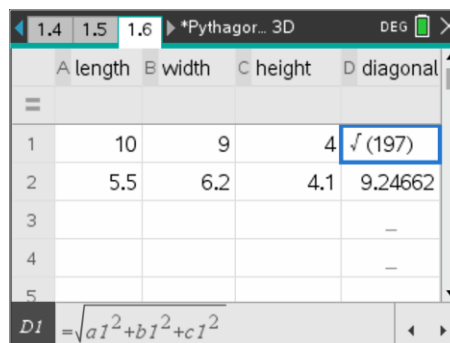
The first set of values are all integers so the result is exact.

The second set of values are decimals so the result is approximate.

	1.4	1.5	1.6	▶ *Pythagor… 3D	DEG
	A length	B width		C height	D diagonal
=					
1	10	9		4	√ (197)
2	5.5	6.2		4.1	9.24662
3					–
4					–
5					

$$D1 = \sqrt{a1^2 + b1^2 + c1^2}$$

Question: 2

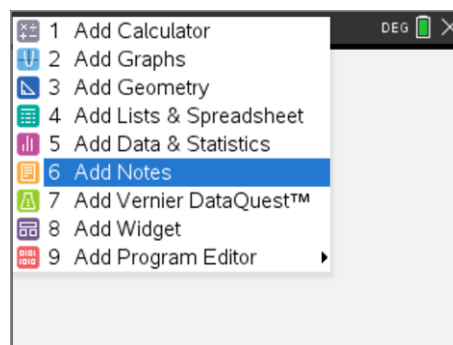What is the formula used to calculate the length of the diagonal?

Question: 3

Complete the table below, which uses decimal values for the dimensions. Enter the values into the Lists&Spreadsheet on page 1.6 to calculate the diagonals (see example below). Write these values in the table and round your answers to two decimal places. Note that when at least one dimension is a decimal, the value of the diagonal will be displayed as a decimal.

Dimensions			List & Spreadsheet page	
length (l)	width (w)	height (h)	Diagonal	Two decimal places
5.5	6.2	4.1	9.24662	9.25
8.4	6.7	4.5		
12.3	8.5	3.4		
20	12	6.5		
26.5	16.5	12.5		

Part C: Using the Notes Application

Another way to perform repeated calculations is to use a Notes page. The following steps show how to set up a Notes page to calculate Pythagoras' Theorem in two dimensions. At the end of this explanation, you will be asked to create a Notes page to calculate Pythagoras' Theorem in three dimensions.

To insert a Notes page, press **[ctrl] + I** and select **Add Notes**.

	DEG
1 Add Calculator	
2 Add Graphs	
3 Add Geometry	
4 Add Lists & Spreadsheet	
5 Add Data & Statistics	
6 Add Notes	
7 Add Vernier DataQuest™	
8 Add Widget	
9 Add Program Editor	▶

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Alternatively, press **[home]** and select the **Notes** icon.

The Notes page allows users to input both text and calculations.

Enter text for the name of the first side.

Then press **[menu] > Insert > Math Box** (or press **[ctrl] +M**) to insert a Math Box.

Input the value **6**.

Press **[ctrl] + [var]** for the **store** command (**sto→**).

Input the variable **a** to represent the first side (avoid using the variables l, w, h, length, width and height as these were used earlier).

Press **[enter]** to finish the calculation.

Repeat to insert a second side with **8** stored in for **b**. Ensure that the text is written on the main screen and the **8→b** is written in a Math Box.

Input the text **hypotenuse**.

Enter the calculation $\sqrt{a^2 + b^2}$ in a Math Box.

Press **[enter]** to calculate the length of the diagonal.

Click in the first two Math Boxes and change the values of the sides. Notice that the value of the hypotenuse automatically changes when the side lengths change.
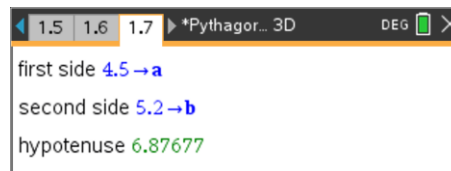
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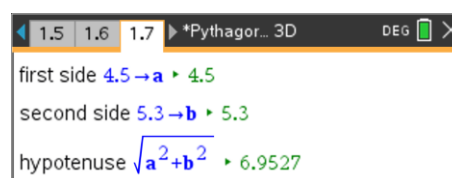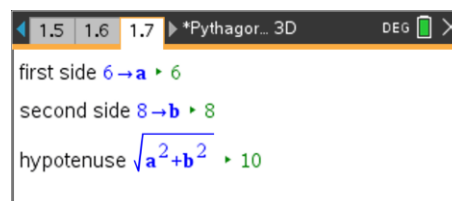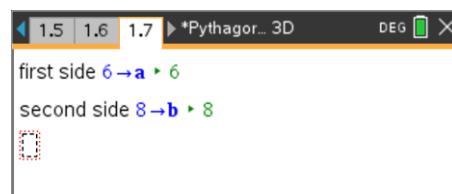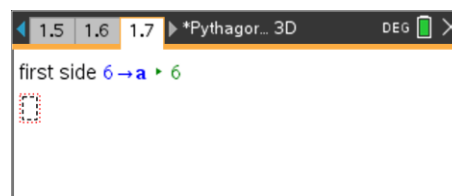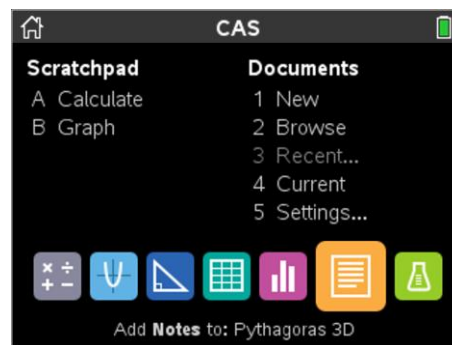
Place the curser in one of the Math Boxes.

Press **menu > Math Box Options > Math Box Attributes**.

In the dialogue box, use the options to **Hide** or **Show** the **Input** and **Output**.
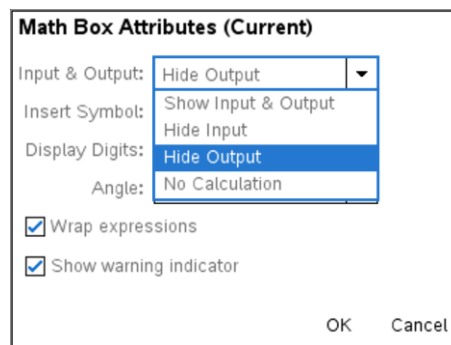
In this screen, the **Output** for the first two Math Boxes and the **Input** for the last Math Box have been hidden.

Extension

You can show the rounded value by inserting another Math Box. Store the formula into the letter **c** as before.

Insert an 'approximately equal to' symbol (≈) as text after the Math Box.
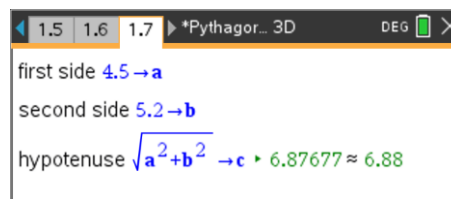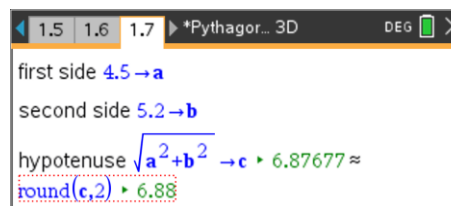(press **[ctrl] + [catalog]** to access the symbol palette).

Press **[ctrl] + M** to insert a new Math Box.

Press **[menu] > Insert > Number > Number Tools > Round** (or select **round(** from the calalog).
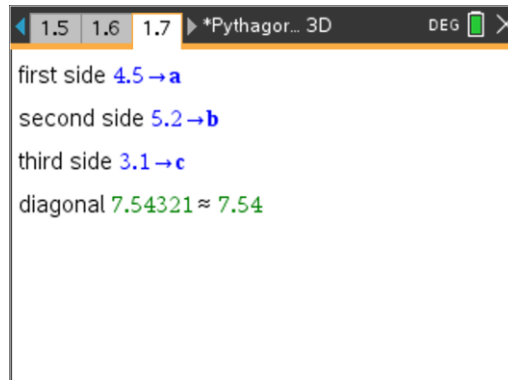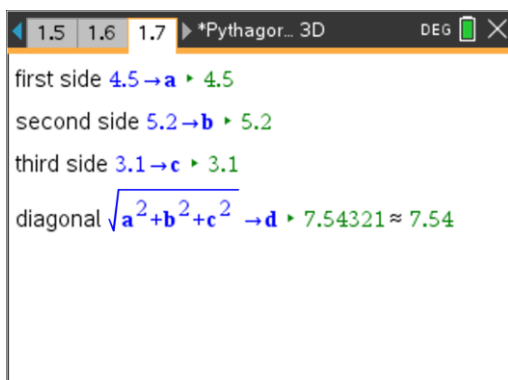
Enter **c,2)** to round the value of **c** to two decimal places and press **[enter]**.

Hide the input to improve the presentation.

Note that this was the method used to create the small Notes page at the bottom of page 1.2.

Amend this Notes page or create a new Notes page to calculate Pythagoras' Theorem in three dimensions.

Question: 4

Use your Notes page to complete the following table. Round your answers to two decimal places.

Dimensions			Notes Page	
length (l)	width 9w)	height (h)	diagonal	2 decimal places
4.2	5.6	3.1	7.65532	7.66
7.9	6.3	5		
8	6	4		
$\frac{10}{3}$	$\frac{5}{2}$	2		
$\sqrt{5}$	$\sqrt{3}$	$3\sqrt{2}$		

Question: 5

Pythagoras' Theorem is generally applied to triangles in two dimensions. Explain why the theorem can be easily adapted by adding a third side to calculate diagonals in three dimensions.

Extension

Pythagorean triples are sets of three integers that adhere to Pythagoras' Theorem. You should already be familiar with Pythagorean triples such as {3, 4, 5}, {5, 12, 13}, {7, 24, 25} and {8, 15 17}. Multiples of these such as {6, 8, 10} and {9, 12, 15} are also Pythagorean triples.

Question: 6

Verify the Pythagorean triples listed above using Pythagoras' Theorem.

Pythagorean quadruples are sets of four integers that adhere to the Pythagorean 3D formula. Two examples are: {3, 4, 12, 13} and {5, 12, 84, 85}.

Question: 7

Using the Calculator page 1.4, the Lists&Spreadsheet page 1.6 or the Notes page you created, enter the first three values of each of the sets above to verify that the fourth value is an integer. Experiment to see if you can find any other Pythagorean quadruples.

Question: 8

Determine a systematic approach for finding Pythagorean quadruples. Give an example of your method.

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