

Using the Document**ParametricMotion.tns**

This calculator file provides a technology tool for visualizing the velocity and acceleration vectors at specific times associated with a particle as it moves along a curve C . The initial curve is defined by

$$x(t) = 4\cos t \qquad y(t) = 3\sin(2t)$$

The velocity and acceleration vectors are computed automatically. The complete curve can be displayed or the moving point alone can be displayed (the geometric trace option is used in this case). The velocity and acceleration vectors can be shown or hidden.

Suggested Applications and Extensions

Use the default curve defined by $x(t) = 4\cos t$, $y(t) = 3\sin(2t)$ to answer the following questions.

Use the slider for t to observe how the velocity and acceleration vectors change as the particle moves along the curve. Show the entire path and the points on the curve as determined by the increment.

1. How is the velocity vector related to the curve?
2. How is the acceleration vector related to the curve?
3. How can the displayed points be used to explain the speed of the particle?
4. How is the velocity vector related to the speed of the particle?
5. Explain how the velocity and acceleration vectors can be used to predict the next point on the curve, as t increases.
6. Is the acceleration vector ever perpendicular to the velocity vector? If so, explain the motion of the particle at that time.
7. Find the velocity and acceleration vectors analytically for $t = \frac{\pi}{4}$. Display the path, and use the slider for t to move the particle to the point on the curve where $t = \frac{\pi}{4}$. Compare your analytical results with the displayed vectors. Explain any discrepancies.
8. Find an equation of the tangent line to the curve at the point where $t = \frac{\pi}{6}$. Explain how the velocity vector of the particle at this time is related to the tangent line.
9. Find all times at which the acceleration vector is $\langle 0, 0 \rangle$. Is there any special characteristic of the velocity vector at these times.

Exploring Motion in the Plane: Velocity and Acceleration

Consider the curve C defined by

$$x(t) = 1 + \int_0^t e^{\cos u} du \quad y(t) = 2 \sin t$$

For $0 \leq t \leq \pi$.

1. Find the velocity and acceleration vectors at time $t = \frac{\pi}{4}$. Confirm your results on the graph. Explain why the velocity and acceleration vectors are pointing in the opposite directions.
2. Use the graph to estimate the time at which the speed of the particle is greatest. Describe the velocity and acceleration vectors at that time.
3. Find, analytically, the time at which the particle is at its highest point on the curve. Find the x - and y -coordinates at that time. Describe the velocity and acceleration vectors at that time, and explain why these are consistent with this position on the curve.
4. Use Page 1.5 to find the straight line distance between the points on the curve that correspond to $t = 0$ and $t = \frac{\pi}{4}$. Find the length of the curve between $t = 0$ and $t = \frac{\pi}{4}$. Compare these two values and explain why they are close in magnitude.
5. Challenge: Describe the concavity of this curve for $0 \leq t \leq \pi$. Do you think there is a relationship between the velocity and acceleration vectors that can determine the concavity of the curve? If so, describe this relationship.

For each of the following curves, describe the velocity and acceleration vectors as t increases through the given interval.

1. $x(t) = \cos^3 t$ $y(t) = \sin^3 t$ $0 \leq t \leq 2\pi$
2. $x(t) = 2\sqrt{t} - 8$ $y(t) = t^2 - 3t - 3$ $0 \leq t \leq 4$
3. $x(t) = 5e^{-t}$ $y(t) = \cos t$ $0 \leq t \leq 2\pi$
4. $x(t) = \ln(t + 1)$ $y(t) = \frac{4t}{t^2 + 1}$ $0 \leq t \leq 10$
5. $x(t) = \sin(2t)$ $y(t) = \tan^{-1} t$ $0 \leq t \leq 2\pi$

Extended Application Questions

1. For a given curve, turn off the Show Path, and turn off Show Vector **a** (the acceleration vector). Use the velocity vector to predict the motion of the particle. Explain your reasoning.
2. For a given curve, turn off the Show Path, and display only the velocity and acceleration vectors. Use these two vectors to predict the motion of the particle. Explain your reasoning.
3. Explain how the displayed points can be used to determine the relative speed of the particle.
4. Explore various curve and try to explain how the velocity and acceleration vectors can be used to determine the concavity of the curve at a point.