Great Expectations

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Student Activity

7 8 9 10 **11** 12

TI-30XPlus MathPrint™ | Investigation | Student | 50 min

Introduction

Sometimes things just don't live up to their 'expectations'. In this activity you will explore three special dice and determine if they live up to their expectations. These dice may be regular in terms of their shape and form; however, each die does not contain the numbers one through to six. The numbers on each die are listed below:

- Red: {3, 3, 3, 3, 3, 6}
- Blue: {2, 2, 2, 5, 5, 5}
- Green: {1, 4, 4, 4, 4, 4}

A simple game is played between two people. Each player selects their own die: Red, Blue or Green. The dice are rolled, the player that rolls the highest number wins.

Question: 1.

Determine the Expected value for each die: Red, Blue and Green.

Question: 2.

Based on your calculations from Question 1, do you think the game is fair?

Creating the dice

The TI-30XPlus MathPrint calculator can generate random numbers.
To begin, press:

random
2nd ! nCr nPr

Select option 2: randint(

The syntax for this command is: **randint(lower, upper)**.

The lower limit is one (1). The upper limit is six (6).

To complete the command press:

A F op
1 2nd , . 6) enter

To continue generating more random numbers keep pressing [enter].

Our 'special' dice do not have the numbers from 1 to 6. A combination of division and the 'integer' command will get us closer to our goal.

The integer command is located in the 'maths' menu. Press:

E
math ▸ 5

Author: P. Fox

The randint(1,6) command needs to be inserted. It can be automatically insterted by copying and pasting!

Arrow up ▲ to highlight the command.

Press enter paste it.

We need to divide our random integers by 6. Press:

% F op
÷ 6) enter

Question: 3.

What sort of numbers are generated by the command combination: int(randint(1,6)/6)?
You may need to press [enter] multiple times to get an idea.

Our simulator is almost complete. The full version to generate random rolls for the red die is as follows:

3 x int(randint(1,6) ÷ 6) + 3

Question: 4.

Explain how: 3 x int(randint(1,6) ÷ 6) + 3 generates random numbers 3 and 6 in the same proportion as the red die.

Question: 5.

What set of random numbers are produced by the command combination:
3 x int(randint(1,6) ÷ 4) + 2 ?

Question: 6.

It's now time to play some games. Select an opponent and your respective colour. (Players must have different colours). Use the corresponding random generator combination for your dice on your calculator whilst your opponent uses their calculator. Remember, you can copy and paste previous commands or simply press enter to recalculate. Record each die outcome, wins and losses in the table below.

Game	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
Red										
Blue										
Winner										
Game	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
Red										
Blue										
Winner										
Game	#21	#22	#23	#24	#25					
Red										
Blue										
Winner										

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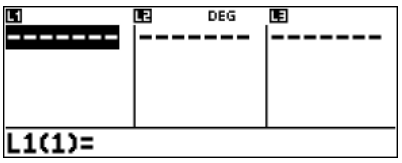
Question: 7.

Based on your relatively small sample of 25 games, do you think the game is fair? Discuss.

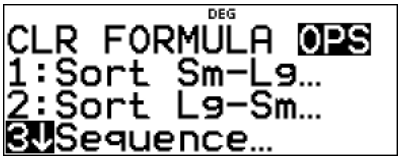
Calculator Simulation

It is possible to simulate 50 games at once!

The TI-30XPlus MathPrint has dynamic lists. To clear all the lists press:



The next step is to create a **sequence** for the red die. The sequence command can be found in the operations (OPS) menu.



The red dice rolls will be stored in List 1. Use the arrow keys to highlight list 1 (L1)

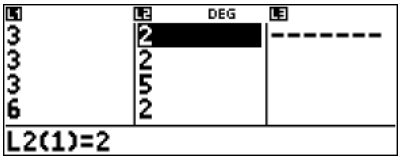
Enter the expression for the red die in the section for "EXPR IN"

3 x int(randint(1,6) ÷ 6) + 3

The sequence can **Start** at 1 and **End** at 50 with a **Step Size** of 1. This will fill List 1 with 50 dice rolls. Select **Sequence Fill** and press **Enter**.
Note: The calculator takes a few moments to generate the entire list.

As more values are entered into the calculator lists it will take longer to display the values, particularly when the lists are based on sequences and formulas. When all three lists are full, this can take a few seconds. Lists appear empty at first until they are re-populated.

An expression also needs to be entered for the blue die. Repeat the sequence process storing the blue dice rolls in List 2.
Notice that the red dice formula remains in the sequence command making it easy to edit and convert to the blue dice.
The screen opposite shows a sample of red dice rolls (List 1) and blue dice rolls (List 2).



It is possible to scroll through the list and see who wins each game, one game at a time. This process can be automated by using some 'clever' calculations:

- When you repeatedly calculate the root of a number between 0 and 1 the result will approach 1 (from below) but never reach it.
- When you repeatedly calculate the root of a number greater than 1, the result will approach 1 (from above) but will never reach it.
- When the red dice wins a larger number appears in List 1 compared to List 2. Calculating List 1 ÷ List 2 would therefore produce a value greater than 1. When red loses the division would produce a result between 0 and 1.

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Consider the situation where red rolls a 6 and blue rolls 2. List 1 ÷ List 2 would result in a 3. Ideally we want this value to be a 1 (win for red). Since $\sqrt{\sqrt{3}} \approx 1.3161$, calculating the integer component only: $int\left(\sqrt{\sqrt{3}}\right) = 1$. The complete formula becomes: $int\left(\sqrt{\sqrt{L1/L2}}\right)$.

To enter the formula in List 3 follow the instructions below.

Whilst in the Data screen, navigate to List 3:

Press:

stat-reg/distr
data

Navigate to "Add/Enter Frmla" (Add / Edit a formula)

A
1

Integer command:

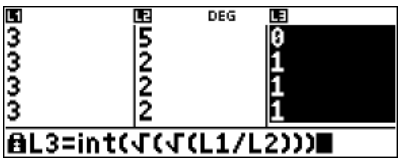
E
math 5

Square-roots:

2nd x^2 2nd x^2

List names and division:

stat-reg/distr A % stat-reg/distr B op op op
data 1 ÷ data 2))) enter



List 3 should will be filled with 0's & 1's.

Scroll through the list to confirm.

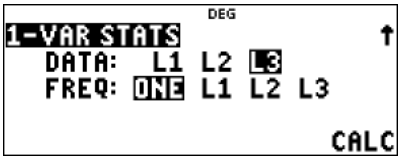
Finally we can use the statistics menu to add up all the values (1's) in List 3.

The statistics menu can find the sum of all the values in a list:

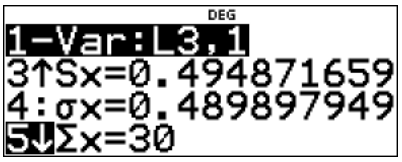
stat-reg/distr B
2nd data 2

Select L3 (List 3) and Frequency as ONE, navigate down to calculate (CALC) and press:

enter



Scroll down to item 5 to see the sum of all the values in the list.
In this case the sum is equal to 30 signifying that the RED dice won 30 games out of 50.



Editing the sequence commands in List 1 and List 2 automatically generates a fresh set of random numbers. To reduce editing time, select the most recent sequence, check the formula and select SEQUENCE FILL. Remember to refresh **both** List 1 and List 2 sequences.

The formula in List 3 does not need to change, it will automatically re-calculate when values in List 1 or List 2 are changed.

Question: 8.

Use the calculator to simulate a total of 100 games of Red vs Blue and record the overall results. From the results decide whether the red or blue dice provides a greater chance of winning.

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Question: 9.

Determine an appropriate formula to generate the numbers on the green die: {1, 4, 4, 4, 4, 4}
Check your formula with your teacher before proceeding.

Question: 10.

Use your formula to generate a **sequence** of 100 dice rolls for the Green die. Simulate 100 games of Blue vs Green and record the overall results. From the results decide whether the blue or green dice provides a greater chance of winning.

Question: 11.

Based on your results to date; is Green more or less likely to beat Red? Discuss.

Question: 12.

Simulate 100 games of Green vs Red and record the overall results. Discuss the outcome.

Question: 13.

Draw probability tree diagrams for each game: Red vs Blue, Blue vs Green and Green vs Red. Use your tree diagrams to explain the results you obtained for Questions 5, 6 and 8.

Question: 14.

Explain why the 'expected' value does not help determine which dice is likely to win.

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