Science Objectives

- Students will determine the frequency conditions that are necessary to produce sound beats.
- Students will relate graphical and algebraic representations of sound beats produced by two tuning forks.
- Students will derive formulas for the wave and beat frequencies in terms of the frequencies of the tuning forks.

Vocabulary

- amplitude
- beat frequency
- empirical
- period
- pressure
- sine wave
- sound beats
- sound wave
- superposition
- tuning
- wave frequency

About the Lesson

- In this activity, students explore graphical and algebraic representations of sound beats produced by two tuning forks and investigate the frequency conditions that are necessary to produce sound beats.
- Students derive formulas for the wave and beat frequencies in terms of the frequencies of the tuning forks.
- Students then apply their understanding of sound beats to problem solving.

TI-Nspire™ Navigator™

- Send out the *Sound_Beats.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Blank sheet of paper and pen or pencil
- Compatible TI Technologies: TI- Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Sound_Beats_Student.doc
- Sound_Beats _Student.pdf

TI-Nspire document

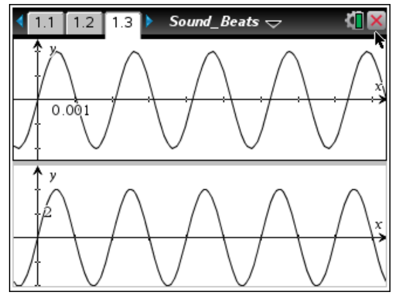
- Sound_Beats.tns

Discussion Points and Possible Answers

Move to pages 1.2 and 1.3.

1. Study the graphs.

   Page 1.3 shows sine wave representations of the sounds produced by the two tuning forks on page 1.2. In these graphs, the *y*-axis represents the pressure changes that produce the sound, and the *x*-axis represents time in seconds.



> **Tech Tip:** Students can use the Text and Calculate tools to enter an expression for frequency in terms of the points on the graph and to find the value of that expression.

Move to pages 1.4 and 1.5.

Have students answer the questions in the .tns file, on the activity sheet, or both.

Q1. Determine which of the graphs represents the sound produced by each of the tuning forks on page 1.2. Explain how you calculated the frequencies of the tuning forks from the graphs.

Answer: The top graph represents the 480 Hz tuning fork; the bottom graph represents the 500 Hz tuning fork. Students should trace each graph to identify the *x*-coordinates of two successive maxima or minima. The difference between these coordinates is the period of the graph. The frequency is the reciprocal of the period. For example, for the second graph,

$$f_2 = \frac{1}{T_2} = \frac{1}{0.0045 - 0.0025} = 500\,s^{-1} = 500\,Hz.$$

> **Tech Tip:** To use the Graph Trace tool, **Menu > Trace > Graph Trace**.

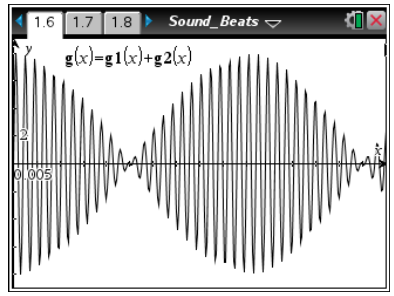
> **Tech Tip:** To use the Graph Trace tool, select **> Trace > Graph Trace**. Students may need to back-out to the main Tools Menu to see the desired menu option.

Q2. Write equations for the two graphs shown on page 1.3. (Remember that the equation for the sine wave representing a sound with frequency *f* is given by $g(x) = A\sin(2\pi fx)$, where *A* is the amplitude of the wave.)

<u>Answer</u>: The equation for the top graph is $g1(x) = 4\sin[2\pi x(500)] = 4\sin(1000\pi x)$.
The equation for the bottom graph is $g2(x) = 4\sin[2\pi x(480)] = 4\sin(960\pi x)$.

Move to page 1.6.

2. Study the graph.



Move to page 1.7.

Have students answer the questions on either in .tns file, on the activity sheet, or both.

> **Tech Tip:** Students should use the **Graph Trace** function to calculate the period of each wave. They should then calculate the frequency of each wave from its period. Sample calculations are shown below.

Q3. Describe any patterns that you see in the graph.

<u>Answer</u>: There are two patterns: a high-frequency wave is modulated by a low-frequency wave, which creates wave packets.

Q4. Determine the frequency of the high-frequency oscillation and of the low-frequency amplitude variation.

<u>Answer</u>: $f_{\text{low}} = \dfrac{1}{0.07449 - 0.02551} = 20.2\,\text{Hz}$ (calculated between zeros of the wave packets);
$f_{\text{high}} = \dfrac{6}{0.05357 - 0.041331} = 490.2\,Hz$ (calculated from the minima of six cycles of the high-frequency wave)

Return to page 1.6.

3. Analyze how the interference of the sound waves produces the beat effect.

The high-frequency wave represents the pitch of the superposed waves (i.e., the pitch of the resulting sound. The low-frequency wave (which modifies the amplitude of the high-frequency wave) represents the changes in loudness of the superposed waves. The frequency of the superposed waves is called the *wave frequency*, f_{wave}. The frequency at which the loudness of the superposed waves seems to change is called the *beat frequency,* f_{beat}.

Introduce the concepts of wave frequency and beat frequency, using the graph on page 1.6.

Describe the physical effects represented by the graph. If possible, demonstrate the phenomenon using tuning forks (or other devices, such as strings on a guitar).

Move to pages 1.8–1.14.

Have students answer the questions in either the .tns file, on the activity sheet, or both

Q5. The graph *g(x)* on page 1.6 represents the sum of the two graphs on page 1.3. Write an equation for the function *g(x)* in terms of the frequencies of the two tuning forks, f_1 and f_2.

Answer: The sine wave for the first tuning fork is described by the equation $g_1(x) = 4\sin(2\pi f_1 x)$, and the sine wave for the second tuning fork is described by the equation $g_2(x) = 4\sin(2\pi f_2 x)$. Therefore, their sum is given by the equation $g(x) = 4\sin(2\pi f_1 x) + 4\sin(2\pi f_2 x)$.

Q6. Use the sum-to-product identity to convert the equation from Question 5 into an equation involving both sine and cosine. Assume that the amplitudes of the two waves are the same. (Do not substitute the given values of f_1 and f_2 into the equation yet.)

Answer: The sum-to-product identity is written as $A\sin(y + z) = 2A\left(\cos\frac{y - z}{2} \bullet \sin\frac{y + z}{2}\right)$.

In this case, $y = 2\pi f_1 x$, $z = 2\pi f_2 x$, and $A = 4$.

Therefore, the equation for the superposed waves is $g(x) = 8\left[\cos\frac{2\pi x(f_1 - f_2)}{2} \bullet \sin\frac{2\pi x(f_1 + f_2)}{2}\right]$.

Q7. The wave produced by adding the two original waves is a sine wave with a varying amplitude. Based on the equation you wrote in Question 6, what is the amplitude of the sine wave in terms of f_1, f_2, and x?

Answer: The amplitude of the sine wave is $A_{wave} = 8\left[\cos\frac{2\pi x(f_1 - f_2)}{2}\right] = 8\cos\left[\pi x(f_1 - f_2)\right]$.

Q8. What is the beat frequency of the combined wave?

Answer: The frequency of the amplitude (loudness) variation is related to the frequency of the cosine term in the overall equation. The loudness of a sound wave is proportional to the square of its amplitude. Therefore, the beat frequency will be proportional to $\left(\cos\left[\pi x(f_1 - f_2)\right]\right)^2$. The cosine half-angle formula is $\left(\cos x\right)^2 = \frac{1 + \cos(2x)}{2}$. Substituting the relevant values yields $f_{beat} \propto \frac{1 + \cos\left[2\pi x(f_1 - f_2)\right]}{2}$. Thus the beat frequency is equal to the absolute value of the difference between the frequencies of the two sounds.

Q9. The pitch of the resultant wave is related to the frequency of the wave. Write an expression for the frequency of the sine wave.

Answer: The equation for a sine wave is $g(x) = A\sin(2pfx)$, where I is frequency.

Therefore, the frequency of the combined wave is $\frac{f_1 + f_2}{2}$.

Q10. Substitute the values of f_1 and f_2 into the equation you derived in Question 6. Write the equation for the superposition of the waves in terms of *x*. Use the *Calculator* application on page 1.13 to confirm your equation.

Answer: $g(x) = 8\left[\cos\frac{2\pi x(20\ Hz)}{2} \bullet \sin\frac{2\pi x(980\ Hz)}{2}\right]$, or $g(x) = 8\left[\cos(20px) \bullet \sin(980px)\right]$.

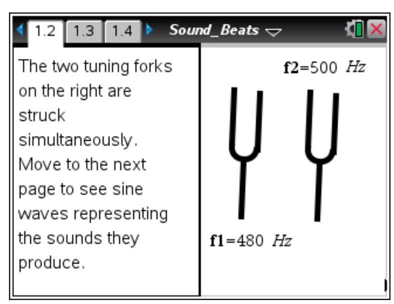
> **Tech Tip:** Students can use the **tCollect** command in the *Calculator* application to confirm that
> $8\left[\cos(20px) \bullet \sin(980px)\right] = 4\sin\left(1000px\right) + 4\sin(960px)$.
> (The right-hand side of this equation is obtained by substituting the given values of A, f_1, and f_2 into the equations for the individual sound waves.) Note: Students must have **TI-Nspire CAS** technology to use the **tCollect** feature.

Q11. Use the expressions you derived in questions 8 and 9 to determine the beat frequency and the wave frequency for the superposed waves on page 1.6. Do your results agree with the empirical results you obtained in Question 4?

Answer: The beat frequency is given by $f_{beat} = |f_1 - f_2| = 20\ \text{Hz}$. This agrees to within 1% with the empirically determined value of 20.2 Hz. The wave frequency is given by $f_{wave} = \frac{f_1 + f_2}{2} = 490\ \text{Hz}$. This agrees to within 0.04% with the empirically determined value of 490.2 Hz.

Return to page 1.2.

4. Change the frequency values for the two tuning forks.
Students should explore how changing the frequency of a tuning fork affects the graphs of the sound waves and the graph of the sound beats.



Tech Tip: Students may have some difficulty selecting the frequency value. Students should tap twice, and get cursor first before trying to change the values. Remind students that they can select undo if they accidently delete the entry box while trying to change the value.

Move to pages 1.15–1.17.

Have students answer the questions in either the .tns file, on the activity sheet, or both.

Q12. How does the graph of the sound wave (page 1.3) change when you increase the frequency of a tuning fork? What happens when you decrease the frequency of a tuning fork?

Answer: When frequency increases, the graph's period decreases (the wave peaks become closer together). When frequency decreases, the graph's period increases (the wave peaks become further apart).

Q13. What happens to the superposition of the sound waves (page 1.6) when you increase or decrease the frequencies of the tuning forks?

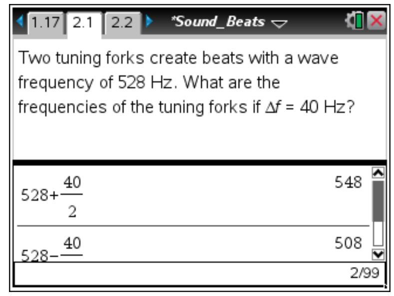
Answer: As the frequency of the second tuning fork becomes much larger than the frequency of the first tuning fork, the wave packets become narrower. Eventually, the sound beats disappear. The same happens when the frequency of the second tuning fork becomes much smaller than the frequency of the first tuning fork.

Q14. What is the maximum frequency difference that will still produce beats? Give your answer in terms of a ratio between the frequency difference, Δf, and the frequency of the first tuning fork, f_1.

Answer: By trial and error, students can find out that the beat pattern disappears when $f_2 > 600\ Hz$ or $f_2 < 360\ Hz$. Thus, one of the appropriate values for Δf £ $120\ \text{Hz}$. A more general answer, given in terms of a ratio, is $\frac{\Delta f}{f_1} \leq \frac{1}{4}$. In most practical applications, it is assumed that beats are produced when $\frac{\text{D}f}{f_1} \approx 0.1$.

Move to page 2.1.

5. A *Calculator* application is provided for computations.



Have students answer the questions in either the .tns file, on the activity sheet, or both.

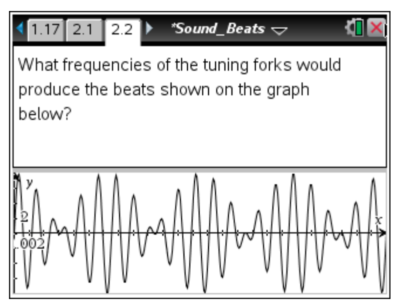
Q15. Two tuning forks create beats with a wave frequency of 528 Hz. What are the frequencies of the tuning forks if $\Delta f = 40\text{ Hz}$?

Answer: Based on the given information, $f_1 - f_2 = 40\text{ Hz}$, *and* $\frac{f_1 + f_2}{2} = 528\text{ Hz}$. *Solving for* f_1 and f_2 *yields* $f_1 = 548\,\text{Hz}$ *and* $f_2 = 508\,\text{Hz}$.

Tech Tip: Students may also use the simultaneous equation solver function, **simult()**, command in the *Calculator* application, to solve the equations.

Move to page 2.2.

6. Use the **Graph Trace** tool to determine wave frequency and beat frequency from the graph. (You can select ctrl tab to move between applications on the screen.) Then answer the question. You may wish to insert a *Calculator* application to check your calculations.



Tech Tip: To insert a *Calculator* application, select **Doc > Insert > Calculator.**

Tech Tip: To insert a *Calculator* application, select **+ > Calculator.**

Q16. What frequencies of the tuning forks would produce the beats shown on the graph?

Answer: Based on the graph, the beat frequency is $\Delta f = \frac{1}{0.04375 - 0.03125} = \frac{1}{0.0125} = 80\,\text{Hz}$.

The pitch frequency can be found by measuring the period of the wave:

$f_{wave} = \frac{1}{0.022619 - 0.020238} = 420\,\text{Hz}$. Thus, $f_1 = 380\,\text{Hz}$ and $f_2 = 460\,\text{Hz}$.

TI-Nspire Navigator Opportunities

Use TI-Nspire Navigator to capture screen shots of student progress and to retrieve the file from each student at the end of the class period. The student questions can be electronically graded and added to the student portfolio.

Wrap Up

When students are finished with the activity, pull back the .tns file using TI-Nspire Navigator. Save grades to Portfolio. Discuss activity questions using Slide Show. Make sure the concept of sound beats is firm in their understanding.

Assessment

- Formative assessment will consist of questions embedded in the .tns file. The questions will be graded when the .tns file is retrieved. The Slide Show will be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test.