Paving Problem

Teachers Notes and Answers

TI-Nspire™   Activity   Student   30 min

ACMNA213 - Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Introduction

Three different landscape garden companies: Mow Town, M.T.Potts and Compost Happens specialise in paving. They are all currently working on a job that requires them to lay pavers around square garden beds of various sizes. Your task is to write a formula for each landscaper that determines the number of pavers required for each job.

Garden 1 – Mow Town Landscapers

Mow Town is an old fashioned landscape company that like to lay their pavers in a specific order.

Open the TI-Nspire file: "Garden Problem 1"

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There are two sliders on the page:

      Garden = side length of garden bed.

      Step = Demonstrates order in which pavers are laid.
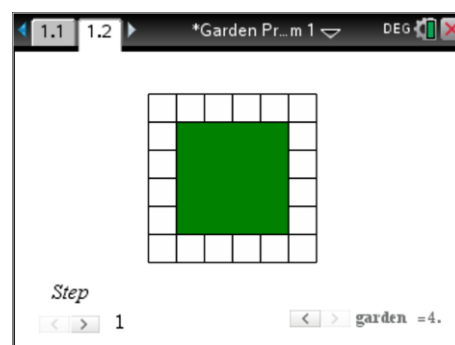
Colours:

      Green = Garden bed

      White = Where pavers will be laid

      Yellow = Pavers currently being laid

      Orange = Pavers laid previously

Start with a 1m x 1m garden (garden = 1) and step gradually through the laying process, then advance to the 2m x 2m garden (garden = 2) and step through the laying process. Continue with this procedure until you have a good understanding of how Mow Town lay their pavers, then answer the questions below.

Question: 1

      How many pavers are laid first, regardless of the garden bed size? (Step 1)

      Answer: 4 Pavers, one on each corner.

Question: 2

      Determine a rule that relates the garden size (x) to the quantity of pavers (y).

      Answer: While numerous variations for the rule can be established, the one that best reflects the method used by Mow Town would be: $y = 4x + 4$

Question: 3

      Explain what part of your rule addresses the quantity of pavers that are laid second. (Step 2)

      Answer: "$4x$" refers to four sides of the square with x pavers on each side to be laid.

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Question: 4

Use your rule to determine how many pavers would be required for a garden bed that is 12m x 12m.

Answer: $y = 4 \times (12) + 4$ (52 pavers)

Question: 5

Mow Town have 40 pavers. What is the largest square garden bed that they can surround? Explain how you determine your answer.

Answer: $40 = 4x + 4$ (9m x 9m garden bed). Answers may vary for the methods used from direct application of the rule to deconstructing the application. "Take out the four corners first leaves 36 paves, there are 4 sides therefore $36 \div 4 = 9$..."

Garden 2 – M.T.Potts

Open the TI-Nspire file: "Garden Problem 2"

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This document is set out the same as previous problem, however M.T.Potts have a different approach to laying their pavers. Your task is to once again explore, step by step, garden by garden, the method used to lay the pavers and an equation that reflects the approach.
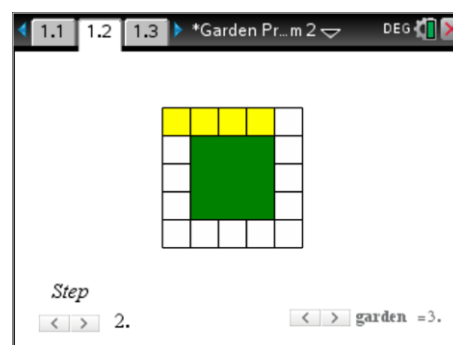
Question: 6

How many pavers are laid first, and how is this related to the size of the garden?

Answer: 'n + 1' pavers are laid first. The side of the garden bed (n) is laid plus an additional tile to cover just one of the corners, hence n + 1 pavers laid first.

Question: 7

How many times is the above (Question 6) process repeated?

Answer: The process is repeated 4 times, once on each side.

Question: 8

Write and equation that relates to the approach applied by M.T.Potts landscapers and explain the relationship.

Answer: $y = 4(n + 1)$ Combining answers from questions 6 & 7, n + 1 pavers are laid on each of the four sides of the garden bed.

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Garden 3 – Compost Happens

Open the TI-Nspire file: "Garden Problem 3"

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Compost Happens do not have a specific order associated with laying pavers however they have a unique method of determining the quantity of pavers required. Shown opposite is "Step 2" for the 4m x 4m solution.

Once again use the step and garden sliders to explore the approach taken by Compost Happens, specifically the method used to determine the quantity of pavers required.
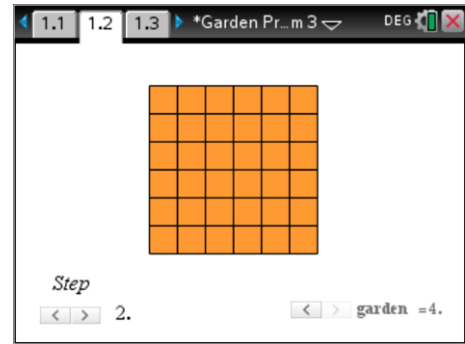
Question: 9

Determine an expression for Step 2 of Compost Happens approach to estimating the quantity of pavers.

Answer: $y = (n+2)^2$ The overall width is 2 pavers longer than the width of the actual garden bed. Note that this includes 'paving the garden bed' ... this quantity must be subtracted later.

Question: 10

Determine the equation derived by Compost Happens for the quantity of pavers required and explain how the company establishes this estimate.

Answer: $y = (n+2)^2 - n^2$ Subtracting the garden bed from the total area leaves just the area to be painted by the gardiner.

Connecting Rules

Question: 11

The three landscape companies each use a different equation that generates exactly the same results. Show that the three equations are actually the same.

Answer: All rules can be written in expanded form, the same as the one used for Mow Town.

$y = 4(n+1) = 4n+4$ - Expanding the formula used by M.T.Potts gives the formula for Mow Town.

$y = (n+2)^2 - n^2 = 4n+4$ - Expanding and simplifying gives the same rule again.

Note: Students may use substitution to show that the rules are the 'same' for the garden sizes in this task, however the algebraic solution shows that this will always be true.

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Open the TI-Nspire file: "A New Garden"

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A new garden bed and paving structure has become popular. Use the garden slider to see how the pavers are set out for the different size garden beds.
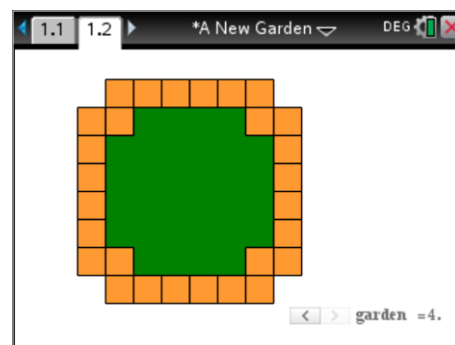
Question: 12

The three landscape companies need to develop paving formulas for the New Gardens. Consider the approaches taken by Mow Town, M.T.Potts and Compost Happens and apply them to the new paving. Determine equations for each landscape company to use and show that the equations are equivalent.

Answer:

Mow Town laid the pavers on the corner of the garden first; this could take on two likely possibilities:

$y = 4(n+2) + 4$ ... this solution sees the first four pavers as those that are stepped in.

$y = 4n + 12$ ...this solution sees 3 paves laid first on each corner, followed by 'n' pavers on each side..

M.T.Potts laid pavers along one complete side; then moved onto the next side. This would most likely yield the equation: $y = 4(n+3)$ One way of visualising this problem is to consider placing the inner corner pavers into the gaps left around the outside of the garden making the problem very similar to the first.

Compost Happens imagined the entire area then subtracted the green area. This could lead to problems if pavers are moved around. The simplest solution would be: $y = (n+4)^2 - (n+2)^2$ ... this approach imagines the four corners extending into the garden being moved to the outside and replacing them with grass.

A more complicated approach would be to slide the sides inwards creating the same garden bed as before:

$y = (n+2)^2 - n^2 + 8$ but then adding on the 8 additional pavers each time.

Expanding and simplifying each expression (above) shows that all of the above rules are the same.

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