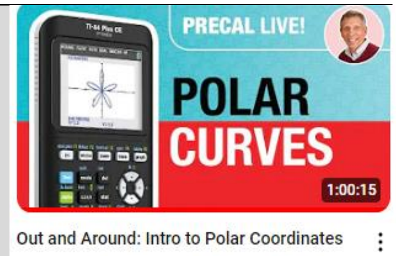
Thursday Night Precalculus Series
March 7, 2024

In this AP Precalculus Live session, we will explore polar coordinates and polar functions.



About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
    - https://www.youtube.com/live/qfUXrTmqq9c?si=BALAFhecP_VzhqzQ
    - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves exploring polar coordinates and the features of the graphs of polar functions, such as:
    - Plotting points.
    - Expressing a complex number in polar form
    - Graphing polar functions.
    - Determining intervals on which the radius increases or decreases.
    - Determining rates of change.
- Students should be able to use the TI-Nspire to verify these features of a polar function.
- **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

AP Precalculus Learning Objectives

- 3.13.A: Determine the location of a point in the plane using both rectangular and polar coordinates.
- 3.14.A: Construct graphs of polar functions.
- 3.15.A: Describe characteristics of the graph of a polar function.

Source: AP Precalculus Course and Exam Description, The College Board

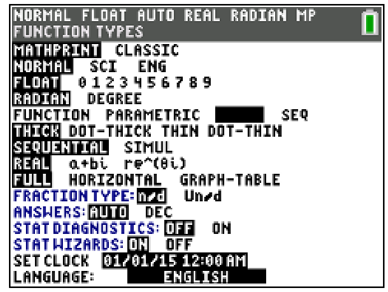
Materials:

Student document
- Precal_problems_03_07

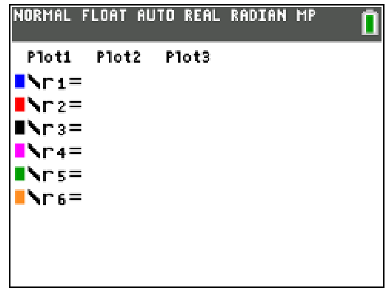
Teacher document
- Precal_problems_solutions_03_07

YouTube
https://www.youtube.com/live/qfUXrTmqq9c?si=BALAFhecP_VzhqzQ
- ***Documents and materials can be downloaded from this site.***

Introduction – Polar Basics



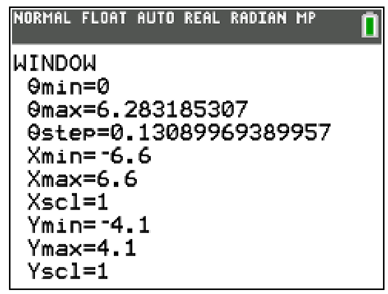
Technology Tip: Change the graphing mode to Polar. Select [mode] and then select POLAR on the 5th line.



Your [y=] key should now show radius function inputs with a new independent variable of θ. Your [X,T,θ,n] key will be θ by default. The window now includes θmin, θmax, and θstep. θ step is $\frac{\pi}{24} \approx 0.13$ by default.

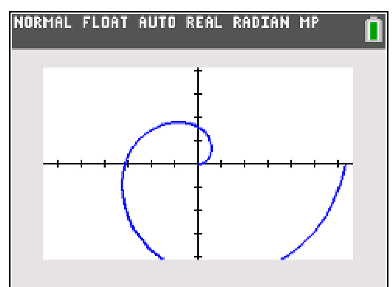
Class Discussion:

Why does the calculator use $\frac{\pi}{24} \approx 0.13$ *by default?*

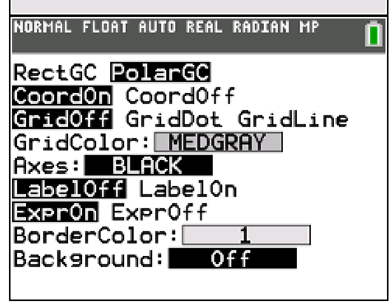


Possible Answers: This step value will naturally take us to the nice rational multiples of π.

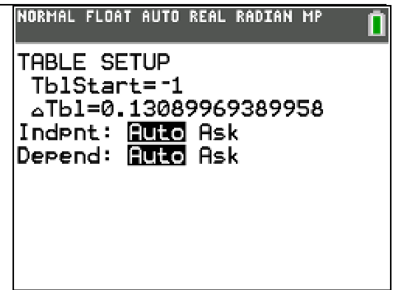
Graph $r = \theta$. Use Trace to observe values of r and θ.

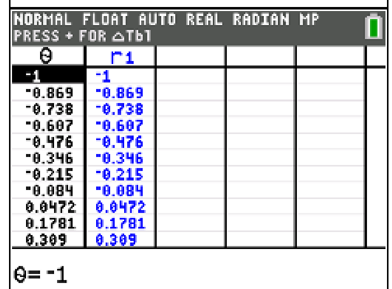


Technology Tip: Trace can be set up to observe r and θ values or x and y values. Select format ([2nd][zoom]) and PolarGC for r and θ values.

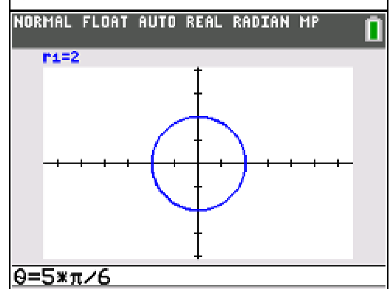


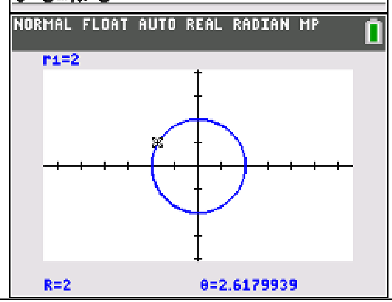
Notice that table ([2nd][GRAPH]) shows r and θ values. Use the Table Setup ([2nd][window]) to change ΔTbl to $\frac{\pi}{24}$.





Graph $r = 2$. Select Trace and ask for a particular θ value, such as $\frac{5\pi}{6}$, by typing $\frac{5\pi}{6}$. This will locate that point on the circle.

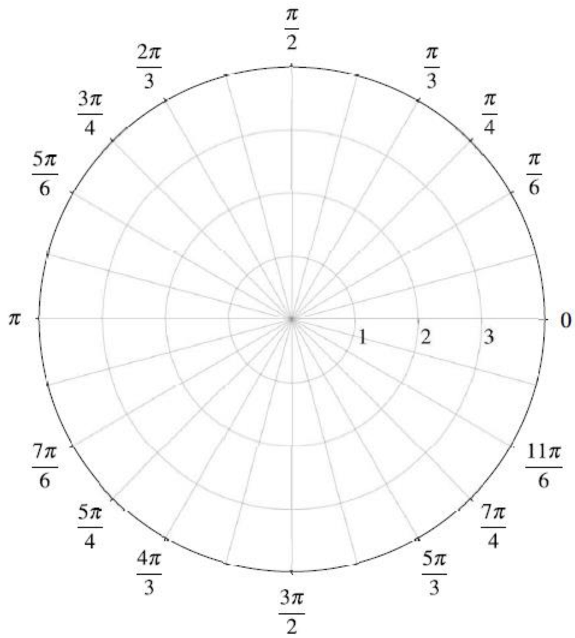




Technology Tip: Making θstep smaller than the default will make the graph more precise. Trace values will be smaller increments. Making θstep larger than the default decreases precision.

Problem 1. (a) – (d)

Plot the points whose polar coordinates are given.



(a) $\left(2, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{\pi}{4}\right)$

(c) $\left(3, -\frac{2\pi}{3}\right)$ (d) $\left(-2, -\frac{\pi}{6}\right)$

Class Discussion:

How do you plot polar points? Do you find the θ first, then locate the r? Or do you locate the r and then sweep around that circle an angle of θ?

Possible Answers: The polar pair is (r, θ). It is probably easier to locate the angle θ, then locate the r, especially if the r-value is negative.

Class Discussion:

Are polar coordinates unique for a specific point?

Possible Answers: No, they are not. For example, $\left(2, \frac{5\pi}{6}\right)$ and $\left(-2, -\frac{\pi}{6}\right)$ are the same point on the polar graph.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 2. (a) & (b)

Convert the polar coordinates to rectangular coordinates.

(a) $\left(\sqrt{2}, \frac{5\pi}{3}\right)$ (b) $\left(-2, -\frac{\pi}{6}\right)$

Class Discussion:

The y-coordinate for 2 (a) is written as $y = -\sqrt{\frac{3}{2}}$. *Could this also be written as* $y = -\frac{\sqrt{6}}{2}$?

Possible Answers: Yes, those values are equivalent.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 3. (a) & (b)

Convert the rectangular coordinates to polar coordinates.

(a) $\left(2, 2\sqrt{3}\right)$ (b) $\left(-1, 2\right)$

Note: The error in the video for 3 (b) is corrected.

Class Discussion:

We frequently use the formula $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ *to find* θ*. The range of the inverse tangent function is* $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$*. When do students need to add* π *to have the correct angle?*

Possible Answers: Consider the quadrant in which the point in rectangular form is located. If the point is in Quadrant II or III, π should be added to the inverse tangent value.

Sample Solution:

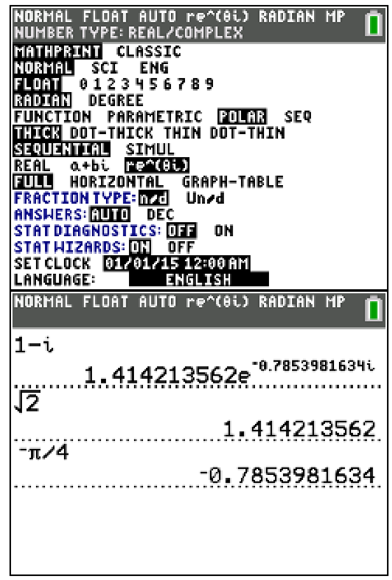
Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 4.

Express the complex number $1-i$ in the polar form $(r\cos\theta)+i(r\sin\theta)$.

<u>Sample Solution:</u>

Refer to the Teacher Solutions Document for the full solution to this problem.



<u>Technology Tip:</u> Select Mode and on the 8th row, select $re^{\wedge}(\theta i)$. This is a polar form using an exponential.

The complex number i is [2nd] [.]

Enter the complex number $1-i$.

<u>Class Discussion:</u>

Can we use this form of the complex number to check our conversions from rectangular to polar?

<u>Possible Answers:</u> Yes, the calculator will display the r and θ. The r and θ will display as decimal values.

Problem 5. (a) – (c)

Create a table of values to sketch each polar graph. Use technology to check your work.

(a) $r=1+\cos\theta$

(b) $r=3\sin(2\theta)$

(c) $r=\theta,\ \theta\geq 0$

<u>Sample Solution:</u>

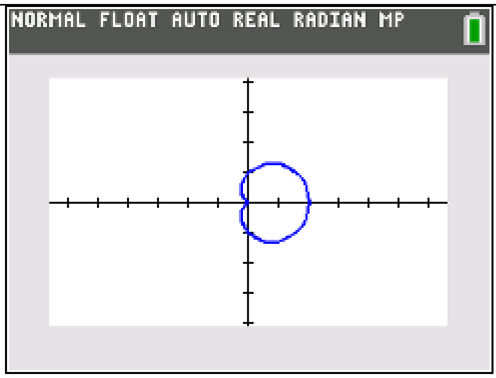
Refer to the Teacher Solutions Document for the full solution to this problem.

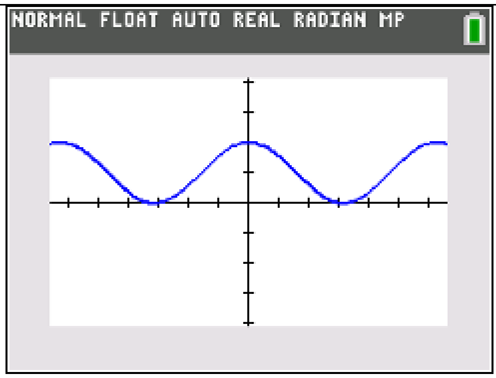
<u>Class Discussion:</u>

Can we use the graph of the rectangular function to graph the polar function?

<u>Possible Answers:</u> Yes, let's look at both graphs and consider the connections.

$r=1+\cos\theta$ $y=1+\cos x$





At $\theta = 0,\ r = 2$ which is a maximum value on the polar graph.
At $\theta = \pi,\ r = 0$ which is a minimum value on the polar graph. That point on the polar graph is at the pole (the origin.)

At $x = 0,\ y = 2$ which is a maximum value on the rectangular graph.
At $x = \pi,\ y = 0$ which is a minimum value on the rectangular graph.

Problem 6. (a) & (b)

Consider the polar function $r(\theta) = \cos\left(\frac{\theta}{2}\right)$ for $0 \le \theta \le 4\pi.$

(a) Graph the polar function over the given domain.

(b) Find the average rate of change of r with respect to θ over the interval $0 \le \theta \le \frac{\pi}{2}.$ Is the radius increasing or decreasing over the given interval? Explain your reasoning.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to explore polar functions.
- The graphing application can be used to explore the behavior of a polar function.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI