Orthocentre
Guided Investigation

Teachers Notes & Answers

7 8 9 **10** **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

The centre of a circle, square, rectangle or regular polygon is relatively easy to locate. The centre of a triangle on the other hand is much harder to define. In this investigation you will explore the ‘orthocentre’, one type of triangle centre.

The orthocentre is located at the intersection of the triangle’s altitudes. When we hear the word altitude, we think of height above a surface, where the height is perpendicular to the surface. So the altitude of a triangle is a perpendicular line segment passing through a vertex of the triangle and the opposite side (or extension of the side).

The QR code or URL contains a video that demonstrates how to set up the Graphs application for this investigation. The video relates to the circumcentre, however the processes are very similar.

https://bit.ly/Circumcentre

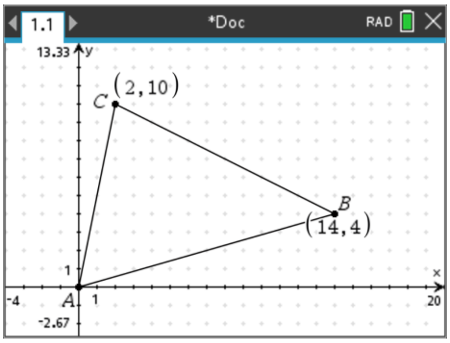
Geometry

Open a New TI-Nspire Document and insert a **Graphs Application**.

Set the window/zoom settings to Quadrant 1 and display a dot grid using the Settings option.

Draw a triangle with vertices:

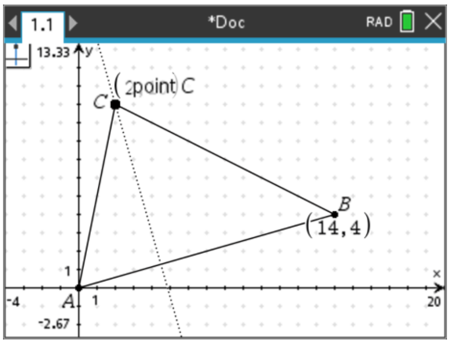
A:(0, 0) B:(14, 4) C:(2, 10)



A perpendicular line can be drawn using the construction tools:

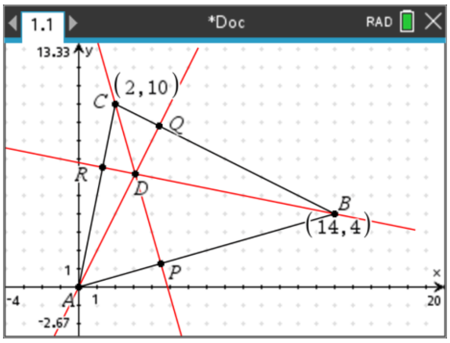
$\boxed{\text{menu}}$ > **Geometry** > **Construction** > **Perpendicular**

Select side A followed by the opposite vertex (C).



Repeat this process to construct all three altitudes, then construct a point of intersection, the orthocentre!

Note: The additional points have been labelled for reference and the lines coloured for visual purposes only.



Author: P. Fox

Question: 1.

Determine the gradient of side AB and hence the gradient of the altitude (PC)

Answer: $\frac{Rise}{Run}=\frac{4-0}{14-0}=\frac{2}{7}$ Perpendicular bisector gradient: $\frac{2}{7}\times m_2=-1$ therefore $m_2=-\frac{7}{2}$

Question: 2.

Determine the equation of the altitude to side AB passing through C. (Line PC)

Answer: $y=-\frac{7}{2}(x-2)+10$ which simplifies to: $y=-3.5x+17$

Tip!

The translational form of a straight line is useful when the gradient and a point are known:

$y=m(x-h)+k$ is a straight line with gradient m passing through the point (h, k).

Remember to use your calculator to check your answers.

Question: 3.

Determine the gradient of side AC and hence the gradient of the altitude (RB).

Answer: $\frac{Rise}{Run}=\frac{10-0}{2-0}=5$ Perpendicular bisector gradient: $5\times m_2=-1$ therefore $m_2=-\frac{1}{5}$

Question: 4.

Determine the equation of the altitude to side AC passing through B. (Line RB)

Answer: $y=-\frac{1}{5}(x-14)+4$ which simplifies to: $y=-0.2x+6.8$

Question: 5.

Determine the gradient of side BC and hence the gradient of the altitude (QA)

Answer: $\frac{Rise}{Run}=\frac{10-4}{2-14}=-\frac{1}{2}$ Perpendicular bisector gradient: $-\frac{1}{2}\times m_2=1$ therefore $m_2=2$

Question: 6.

Determine the equation of the altitude to side BC passing through A. (Line QA)

Answer: $y=2(x-0)+0$ which simplifies to: $y=2x$

Question: 7.

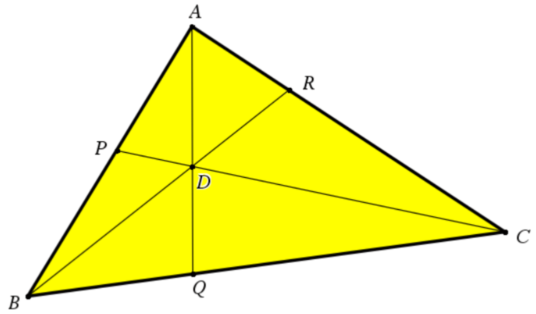
Use simultaneous equations to determine the point of intersection for the altitudes: QA and PC.

Answer: Equations: $y=2x$ & $y=3.5x+17$

$$\begin{aligned}-3.5x+17&=2x\\17&=5.5x\\x&=\frac{34}{11}\end{aligned}\qquad\qquad\begin{aligned}y&=2\left(\frac{34}{11}\right)\\y&=\frac{68}{11}\end{aligned}$$

Author: P. Fox

Extension – Ceva's Theorem



Ceva's theorem states that in order for AQ, BR and CP to be concurrent (meet at a single point D) then the following must apply:

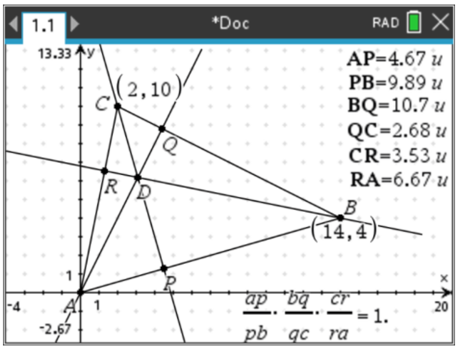
$$\frac{AP}{PB}\times\frac{BQ}{QC}\times\frac{CR}{RA}=1$$

Note: AP represents the distance from A to P.

Use Ceva's theorem to show that the altitudes of the triangle with vertices: A:(0, 0), B:(14, 4), C:(2, 10) are concurrent.

Teacher Notes:

This could be completed as a calculator activity using tools available in TI-Nspire. This can be achieved through various degrees. The most technology sophisticated approach is for students to measure and store lengths accordingly, insert Ceva's theorem as text and then calculate. This result will be completely dynamic so that students can continue to move points A, B and C to see everything update automatically.



Students can apply a simpler approach and just use the calculator to determine the location of points P, Q and R then compute distances using the distance formula. Students should still be encouraged to store distances in variables: AP, PB etc.

The solution provided below summarises a complete by hand solution.

Answers:

Equation for side AB: $y=\frac{2}{7}x$ New calculation

Equation for altitude CP: $y=-3.5x+17$ Previous calculation

Simultaneous Equations for altitude CP and side AB to find point P: $\left(\frac{238}{53},\frac{68}{53}\right)$

Distances: $\overline{AP}=\frac{34\sqrt{53}}{53}$ $\overline{PB}=\frac{72\sqrt{53}}{53}$

Equation for side BC: $y=-\frac{1}{2}x+11$ New calculation

Equation for altitude CP: $y=2x$ Previous calculation

Simultaneous Equations for altitude CP and side AB to find point P: $\left(\frac{22}{5},\frac{44}{5}\right)$

Distances: $\overline{BQ}=\frac{24\sqrt{5}}{5}$ $\overline{QC}=\frac{6\sqrt{5}}{5}$

Author: P. Fox

Equation for side CA: $y = 5x$ New calculation

Equation for altitude RB: $y = -0.2x + 6.8$ Previous calculation

Simultaneous Equations for altitude RB and side CA to find point P: $\left(\frac{17}{13}, \frac{85}{13}\right)$

Distances: $\overline{CR} = \frac{9\sqrt{26}}{13}$ $\overline{PB} = \frac{17\sqrt{26}}{13}$

Finally: $\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = \frac{17}{36} \times \frac{4}{1} \times \frac{9}{17} = 1$ **Note**: Lots of simple and neat simplifications!

Conclusion: By Ceva's theorem the three lines: AQ, BR and CP are concurrent.

Author: P. Fox