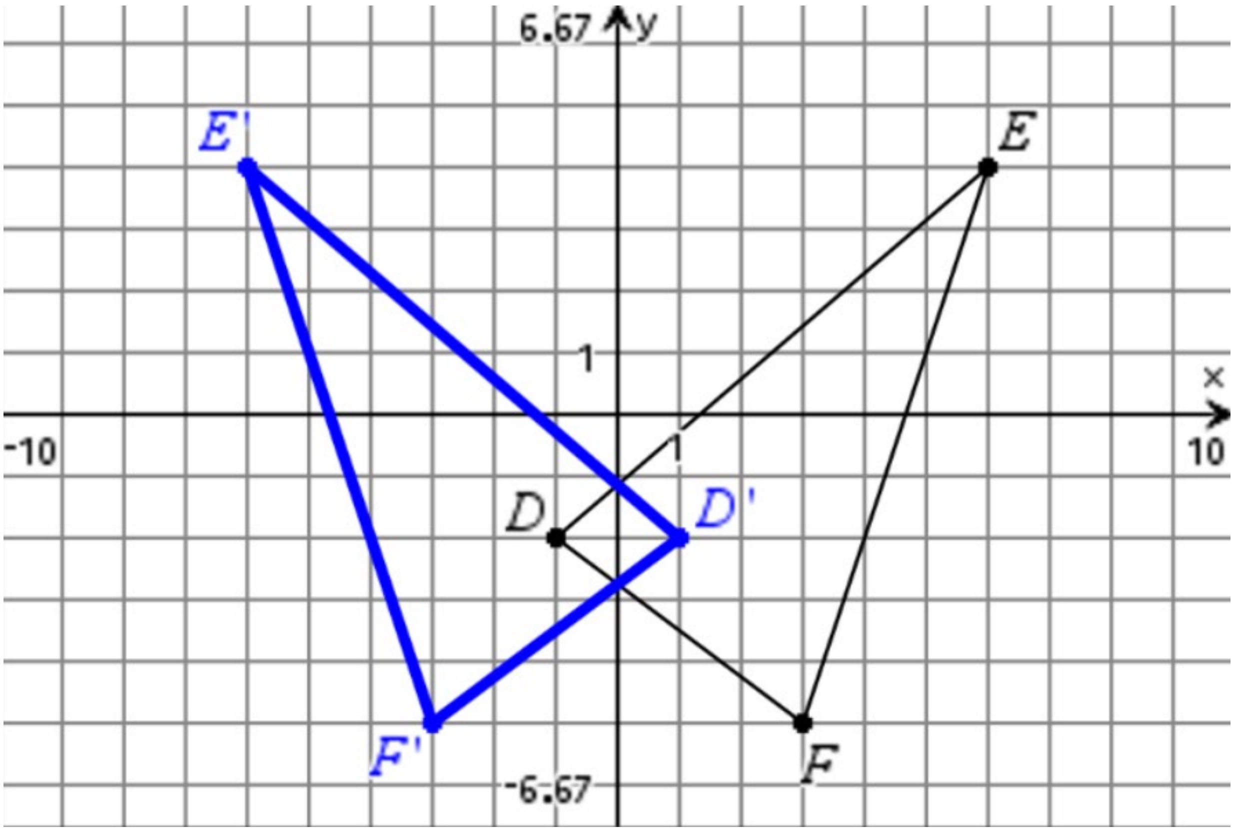
In this lesson, you will be given the opportunity to summarize, review, explore and extend ideas about each of the four transformations: reflections, translations, rotations, dilations.
Use a straightedge to make sketches in the grid supplied.

1. Reflect ΔDEF about the y-axis. Then fill in the blanks with appropriate responses.



a. If $m\angle F = 70^o$, then $m\angle F' = 70^o$

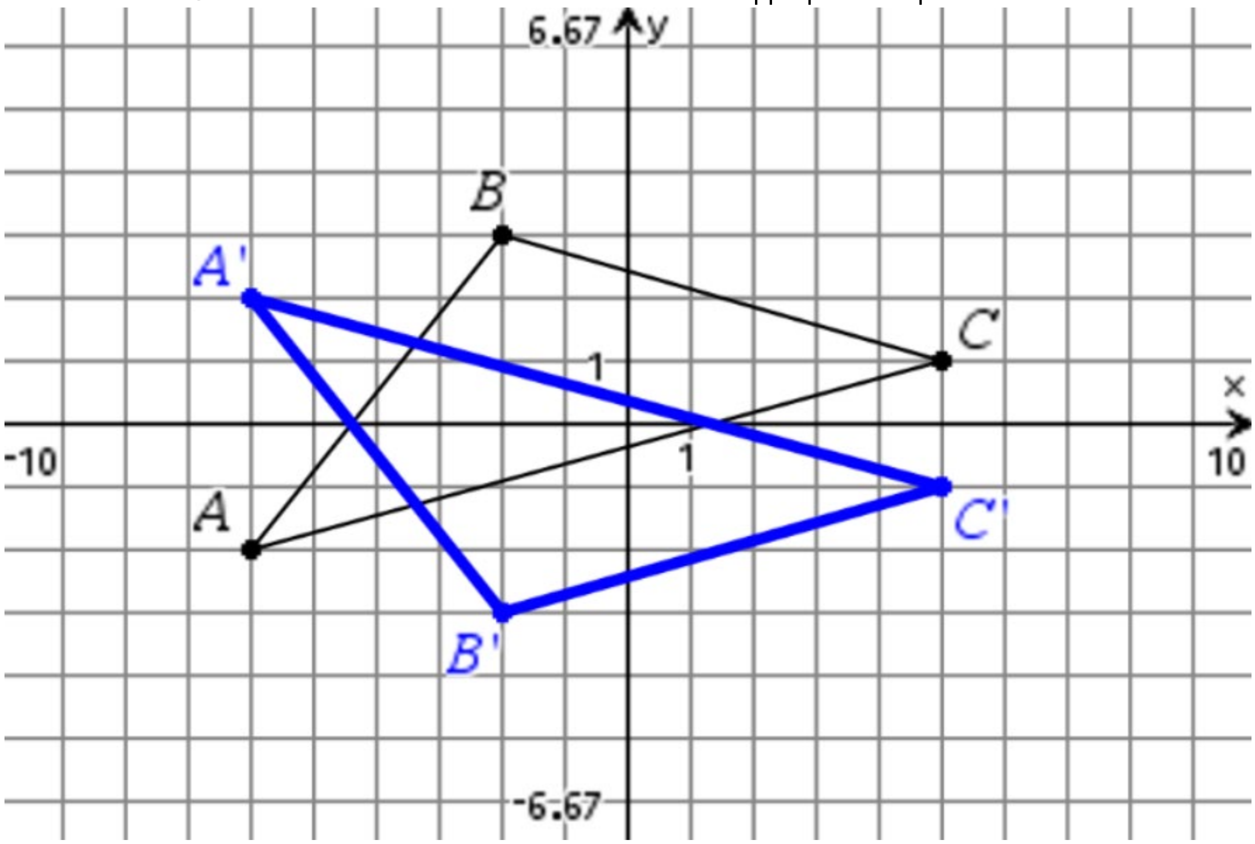
b. if the slope of $\overline{DE} = \frac{6}{7}$, then the slope of $\overline{D'E'} = -\frac{6}{7}$

c. If the coordinates of E are (6, 4), then the coordinates of E' *are* $(-6, 4)$

d. If the area of ΔDEF is 24 sq cm, then the area of $\Delta D'E'F'$ *is* 24 *sq cm*

e. If the coordinates of a point H on ΔDEF are (x, y), then the coordinates of H' are $(-x, y)$

2. Reflect ΔABC about the x-axis. Then fill in the blanks with appropriate responses.



a. If $m\angle A = 35^o$, then $\underline{\underline{m\angle A' = 35^o}}$

b. If BC = 8 cm, then $\underline{\underline{B'C' = 8\ cm}}$

c. If the slope of $\overline{BC} = -\frac{2}{7}$, then the slope of $\underline{\underline{\overline{B'C'} = \frac{2}{7}}}$

d. If the perimeter of ΔABC = 17 in, then the perimeter of $\underline{\underline{\Delta A'B'C' = 17\ in}}$

e. If the coordinates of a point G on ΔABC are (x, y), then the coordinates of G' are $\underline{\underline{(x, -y)}}$

f. If the coordinates of a point H' on $\Delta A'B'C'$ are $\left(3, -1\frac{1}{2}\right)$,

then the coordinates of H are $\underline{\underline{\left(3, 1\frac{1}{2}\right)}}$

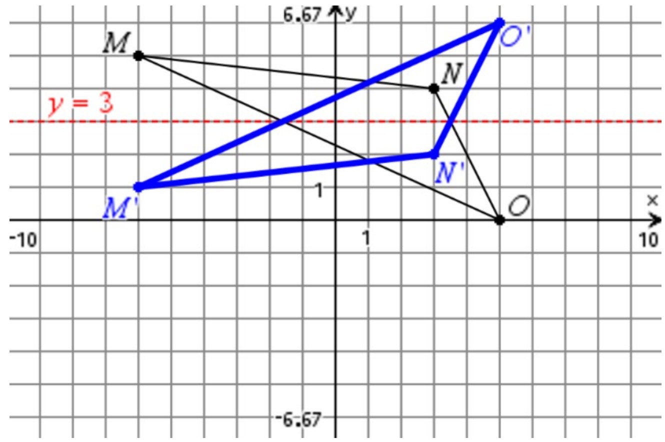
3. Reflect ΔMNO about the line y = 3.

List the coordinates of each of the vertices:

$M: (-6, 5)$ $M': (-6, 1)$

$N: (3, 4)$ $N': (3, 2)$

$O: (6, 0)$ $O': (6, 6)$



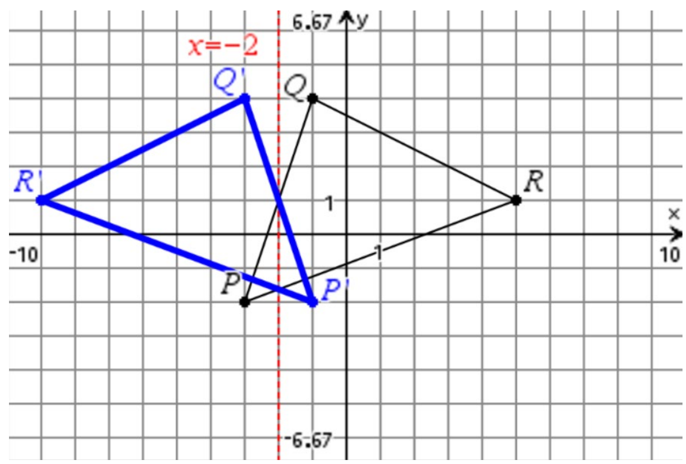
4. Reflect ΔPQR about the line x = – 2.

List the coordinates of each of the vertices:

$P: (-3, -2)$ $P': (-1, -2)$

$Q: (-1, 4)$ $Q': (-3, 4)$

$R: (5, 1)$ $R': (-9, 1)$



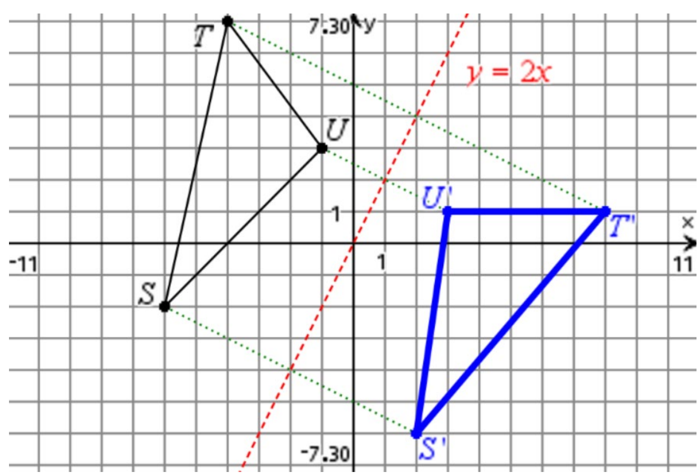
5. Reflect ΔSTU about the line y = 2x.

List the coordinates of each of the vertices:

$S: (-6, -2)$ $S': (2, -6)$

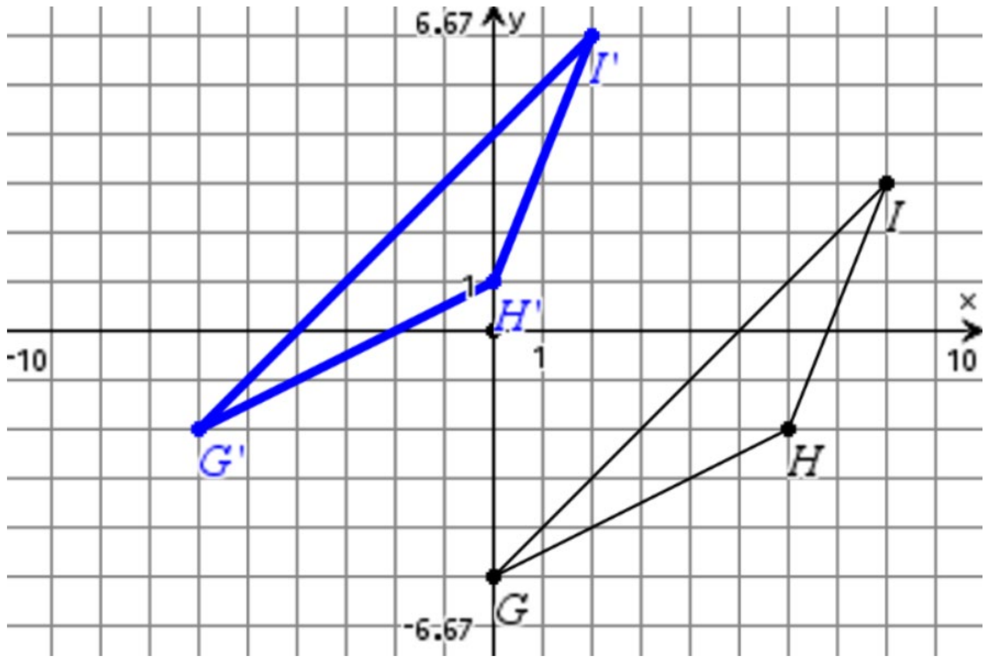
$T: (-4, 7)$ $T': (8, 1)$

$U: (-1, 3)$ $U': (3, 1)$



Hint: the slope of the line $y = 2x$ is 2. The slope of the connected segments, $\overline{SS'}$, $\overline{UU'}$, $\overline{TT'}$, must each be $-\frac{1}{2}$ because they are perpendicular to the line $y = 2x$ (and parallel to each other).

6. Translate ΔGHI up 3 units and to the left 6 units. Then fill in the blanks with appropriate responses.



a. If GH = 9 in, then $\underline{\underline{G'H' = 9in}}$

b. If the perimeter of ΔGHI is 36 cm, then the perimeter of $\underline{\underline{\Delta G'H'I' \ is \ 36cm}}$

c. If the slope of $\overline{HI} = \dfrac{5}{2}$, then the slope of $\underline{\underline{\overline{H'I'} = \dfrac{5}{2}}}$

d. If the coordinates of H are (6, – 2), then the coordinates of $\underline{\underline{H' \ are \ (0, 1)}}$

e. If point A is on ΔGHI and its coordinates are $(3, -2)$, the coordinates of A' are $\underline{\underline{(-3, 1)}}$

f. If point Z' is on $\Delta G'H'I'$ and its coordinates are $(-2, \ 2)$, the coordinates of Z: $\underline{\underline{(4, -1)}}$

g. If the coordinates of a point P on ΔGHI are (x, y), then the coordinates of P' are $\underline{\underline{(x-6, y+3)}}$

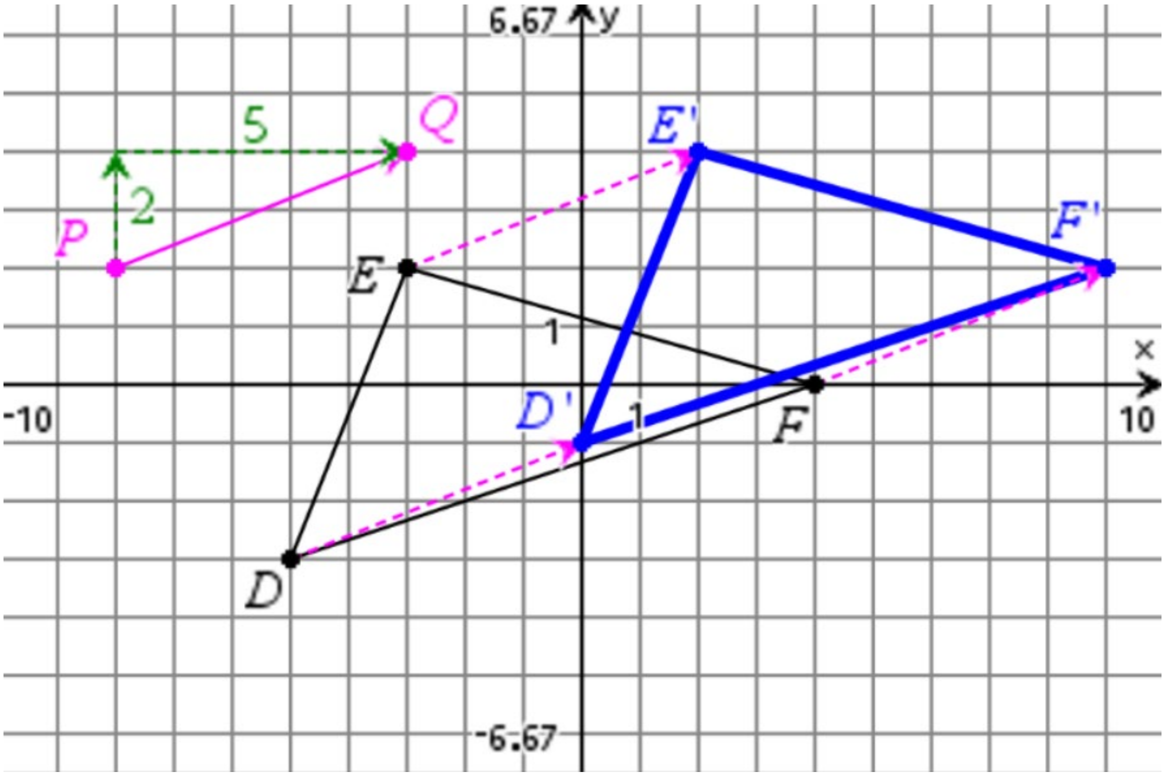
h. Name three sets of parallel segments and list the slope of each:

$\underline{\underline{\overline{GH} \parallel \overline{G'H'} \qquad slope \ is \ \dfrac{1}{2}}}$

$\underline{\underline{\overline{HI} \parallel \overline{H'I'} \qquad slope \ is \ \dfrac{5}{2}}}$

$\underline{\underline{\overline{GI} \parallel \overline{G'I'} \qquad slope \ is \ 1}}$

7. Translate ΔDEF by vector $\overrightarrow{PQ}$.



a. What are the coordinates of $\underline{\underline{D': (0, -1)}}$ $\underline{\underline{E': (2, 4)}}$ $\underline{\underline{F': (9, 2)}}$

b. If point A' is on $\Delta D'E'F'$ and has coordinates $(6, 1)$, the coordinates of A? $\underline{\underline{(1, -1)}}$

c. What segments are parallel to vector $\overrightarrow{PQ}$? $\overline{DD'} \parallel \overline{EE'} \parallel \overline{FF'} \parallel \overrightarrow{PQ}$

What is the slope of each of those segments? $\underline{\underline{\frac{2}{5}}}$

d. Name three other pairs of segments that are also parallel and state their slopes:

$\overline{DE} \parallel \overline{D'E'}$ $\quad$ *slope is* $\frac{5}{2}$

$\overline{EF} \parallel \overline{E'F'}$ $\quad$ *slope is* $-\frac{2}{7}$

$\overline{DF} \parallel \overline{D'F'}$ $\quad$ *slope is* $\frac{1}{3}$

8. Given: ΔDEF is translated to the left 7 units and up 5 units.

a. If D has coordinates (5, 7), what are the coordinates for D'? $\underline{\underline{(-2, 12)}}$

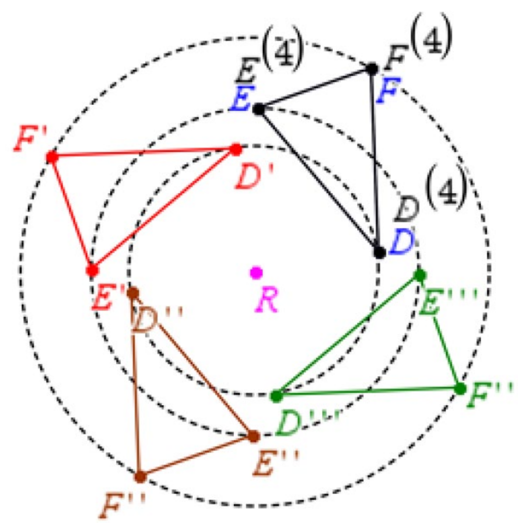
b. If E has coordinate (– 3, – 7), what are the coordinates of E'? $\underline{\underline{(-10, -2)}}$

c. If F' has coordinates (1, 6), what are the coordinates of F? $\underline{\underline{(8, 1)}}$

d. If D has coordinates (x, y), what are the coordinates for D'? $\underline{\underline{(x-7,\ y+5)}}$

e. If E' has coordinates (p, q), what are the coordinates for E? $\underline{\underline{(p+7, q-5)}}$

9. Label the vertices of the images appropriately.



a. Rotate ΔDEF 90° about point R. ($\Delta D'E'F'$)

b. Rotate ΔDEF 180° about point R. ($\Delta D''E''F''$)

c. Rotate ΔDEF 270° about point R. ($\Delta D'''E'''F'''$)

d. Rotate ΔDEF 360° about point R. ($\Delta D^{(4)}E^{(4)}F^{(4)}$)

e. If $m\angle D = 35^o$, then $m\angle D' = \underline{\underline{35^o}}$

f. If EF = 4.5 in, then E''F'' = $\underline{\underline{4.5\ in}}$

g. If the slope of $\overline{ED}$ = – 2, then the slope of $\overline{E'D'}$ = $\underline{\underline{\frac{1}{2}}}$

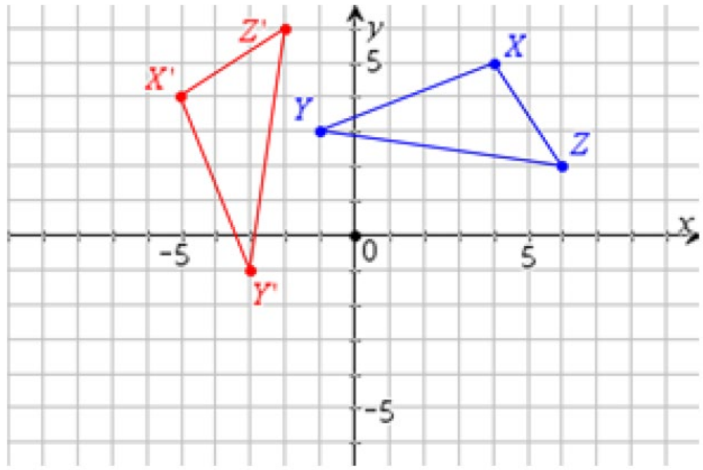
h. If the slope of $\overline{EF}$ = $\frac{2}{3}$, then the slope of $\overline{E''F''}$ = $\underline{\underline{\frac{2}{3}}}$

i. If the perimeter of ΔDEF is 8 in, then the perimeter of $\Delta D''E''F''$ is $\underline{\underline{8\ in}}$

j. If the coordinates of point D are (3, 2), what are the coordinates of:

D': $(2, -3)$ D'': $(-3, -2)$ D''': $(-2, 3)$ $D^{(4)}$: $(3, 2)$

10. Label the vertices of the images appropriately.



a. Rotate ΔXYZ 90° about the origin.

$m(\overline{XY}) = \frac{2}{5}$ $m(\overline{X'Y'}) = -\frac{5}{2}$

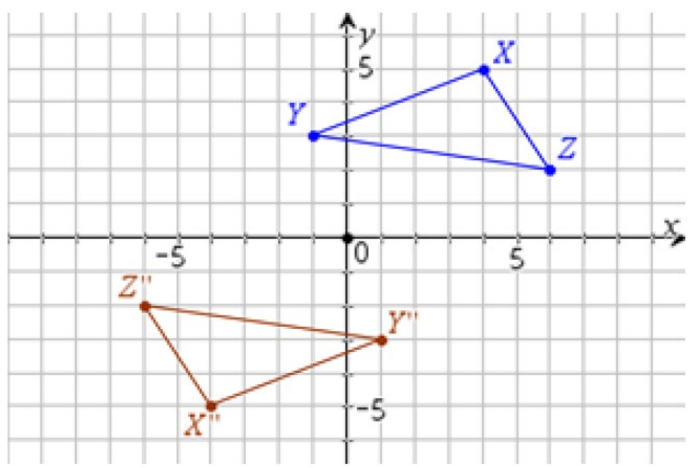
$m(\overline{YZ}) = -\frac{1}{7}$ $m(\overline{Y'Z'}) = \frac{7}{1}$

$m(\overline{XZ}) = -\frac{3}{2}$ $m(\overline{X'Z'}) = \frac{2}{3}$

Fill in the blanks with either ∥ ('is parallel to') or $\perp$ (' is perpendicular to'):

$\overleftrightarrow{XY} \perp \overleftrightarrow{X'Y'}$ $\overleftrightarrow{YZ} \perp \overleftrightarrow{Y'Z'}$ $\overleftrightarrow{XZ} \perp \overleftrightarrow{X'Z'}$

11. Label the vertices of the images appropriately.



b. Rotate ΔXYZ 180° about the origin.

$m(\overline{XY}) = \frac{2}{5}$ $m(\overline{X'Y'}) = \frac{2}{5}$

$m(\overline{YZ}) = -\frac{1}{7}$ $m(\overline{Y'Z'}) = -\frac{1}{7}$

$m(\overline{XZ}) = -\frac{3}{2}$ $m(\overline{X'Z'}) = -\frac{3}{2}$

Fill in the blanks with either ∥ ('is parallel to') or $\perp$ (' is perpendicular to'):

$\overleftrightarrow{XY} \parallel \overleftrightarrow{X''Y''}$ $\overleftrightarrow{YZ} \parallel \overleftrightarrow{Y''Z''}$ $\overleftrightarrow{XZ} \parallel \overleftrightarrow{X''Z''}$

12.a. The corresponding sides of rotated triangles are <u>congruent, that is have the same length.</u>

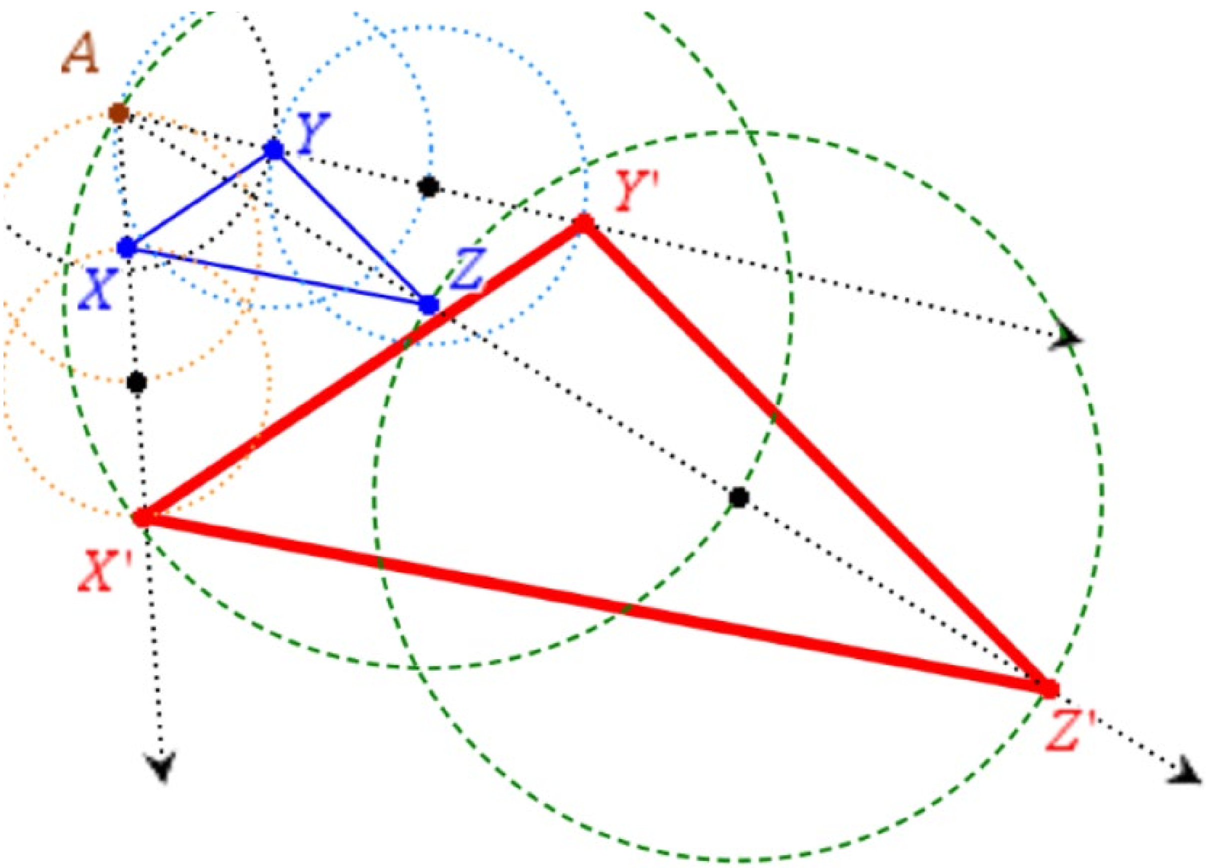
b. The corresponding angles of rotated triangles are <u>congruent, that is, have the same measure.</u>

13. If a triangle is rotated about a point through x°, the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent and the triangles are <u>congruent.</u>

Therefore, a rotation is a <u>**rigid motion**</u> or an <u>**isometry**</u>.

We also say that a rotation is a <u>**distance-preserving**</u> and an <u>**angle-preserving**</u> transformation.

14. **All of the questions in this exercise refer to the dilation that you will do below.**
Dilate ΔXYZ about point A with a scale factor of 3.



a. If $m\angle X = 20^o$, then $m\angle X' = \underline{\underline{20^o}}$

b. If $YZ =$ 8 cm, then $Y'Z' = \underline{\underline{24\,cm}}$

c. If $X'Z' =$ 30 in, then $XZ = \underline{\underline{10\,in}}$

d. If the perimeter of ΔXYZ is 60 cm, then the perimeter of $\Delta X'Y'Z' = \underline{\underline{180\,cm}}$

e. Calculate the following ratios. Write your answers as fractions.

1. $\dfrac{perimeter(\Delta X'Y'Z')}{perimeter(\Delta XYZ)} = \underline{\underline{\dfrac{3}{1}\ or\ 3}}$

2. $\dfrac{area(\Delta X'Y'Z')}{area(\Delta XYZ)} = \underline{\underline{\dfrac{9}{1}\ or\ 9}}$

3. $\dfrac{perimeter(\Delta XYZ)}{perimeter(\Delta X'Y'Z')} = \underline{\underline{\dfrac{1}{3}}}$

f. If the area of ΔXYZ = 72 in^2, then the area of $\underline{\underline{\Delta X'Y'Z' = 648\ in^2}}$

g. What is true about the segments $\overline{XZ}$ *and* $\overline{X'Z'}$? Parallel

h. The slope of $\overline{XY}$ is $-\dfrac{3}{4}$. List another segment and its slope. $\underline{\underline{slope\ of\ \overline{X'Y'}\ is\ -\dfrac{3}{4}}}$

i. If $AX = 10\,cm$, then $AX' = \underline{\underline{30\,cm}}$ *and* $XX' = \underline{\underline{20\,cm}}$

j – o. Calculate the ratios. Write your answers as fractions.

j. $\dfrac{AX'}{AX} = \underline{\underline{\dfrac{3}{1}\ or\ 3}}$

k. $\dfrac{AY}{AY'} = \underline{\underline{\dfrac{1}{3}}}$

l. $\dfrac{XZ}{X'Z'} = \underline{\underline{\dfrac{1}{3}}}$

m. $\dfrac{area(\Delta XYZ)}{area(\Delta X'Y'Z')} = \underline{\underline{\dfrac{1}{9}}}$

n. $\dfrac{m\angle X}{m\angle X'} = \underline{\underline{\dfrac{1}{1}\ or\ 1}}$

o. $\dfrac{m\angle Z'}{m\angle Z} = \underline{\underline{\dfrac{1}{1}\ or\ 1}}$

p. If point A is at the origin, answer the following questions.

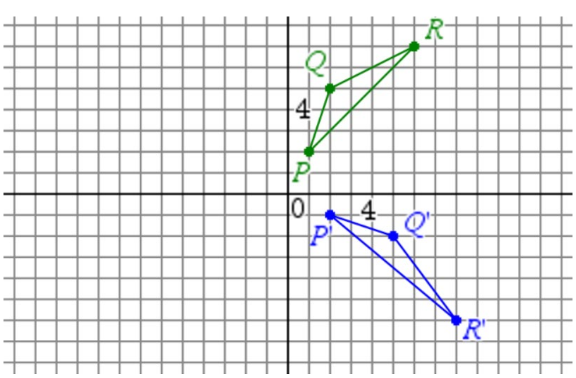
1. If the coordinates of X are $(6,-12)$, then the coordinates of X' are $\underline{\underline{(18,-36)}}$
2. If the coordinates of Z' are $(6,-12)$, then the coordinates of Z are $\underline{\underline{(2,\,-4)}}$
3. If the coordinates of Y are $(-7,11)$, then the coordinates of Y' are $\underline{\underline{(-21,\,33)}}$
4. If the coordinates of X' are $(-18,24)$, then the coordinates of X are $\underline{\underline{(-6,\,8)}}$

q. If point A were to coincide with point X:

1. Which pairs of sides will overlap? $\underline{\underline{\overline{XY}\ \ and\ \ \overline{X'Y'}\quad \overline{XZ}\ \ and\ \ \overline{X'Z'}}}$
2. What is the other pair of sides and what is true about these sides? $\underline{\underline{\overline{YZ}\ \square\ \overline{Y'Z'}}}$

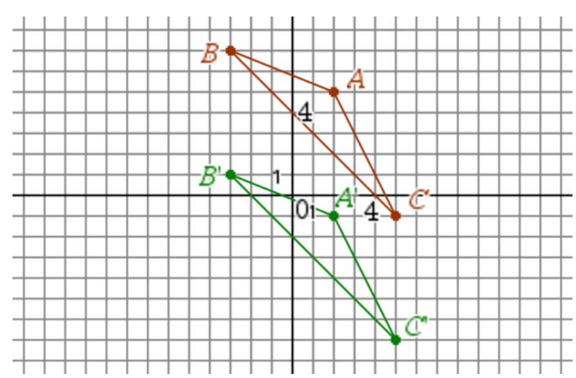
15. In each of the following grids, a triangle was transformed.
State which transformation was done: dilation, reflection, rotation, translation.
And describe what was done: how many units, which direction, about what angle, …

a. pre-image ΔPQR; image $\Delta P'Q'R'$



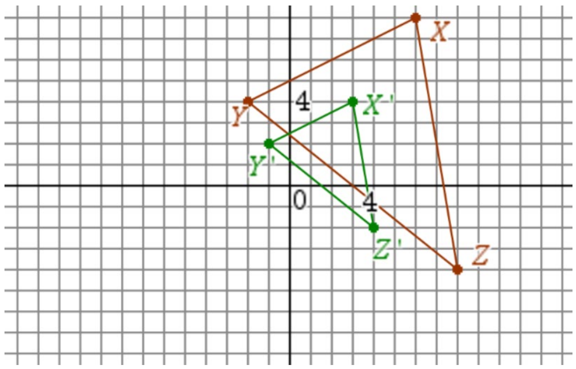
Rotate $\Delta PQR\ \ 270^o$ about the origin.
or Rotate $\Delta PQR\ \ -90^o$ about the origin.

b. pre-image ΔABC; image $\Delta A'B'C'$



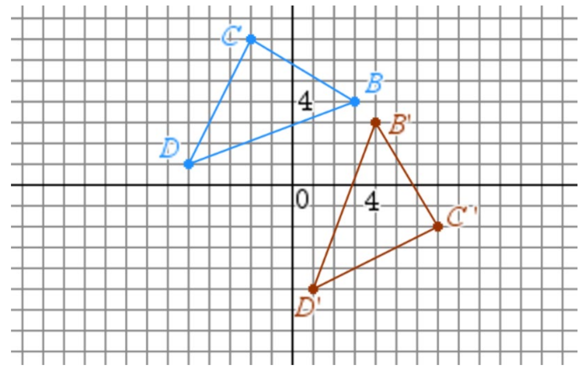
Translate ΔABC down 6 units.

c. pre-image ΔXYZ ; image $\Delta X'Y'Z'$



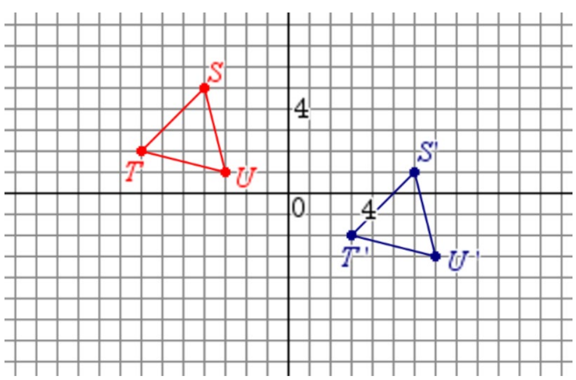
Dilate ΔXYZ about the origin with a Scale Factor of $\frac{1}{2}$.

d. pre-image ΔBCD ; image $\Delta B'C'D'$



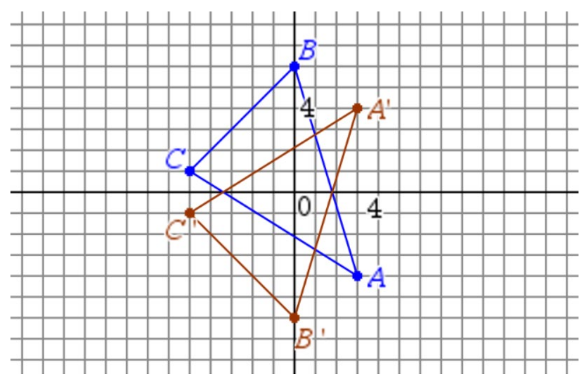
Reflect ΔBCD about the line y = x.

e. pre-image ΔSTU ; image $\Delta S'T'U'$



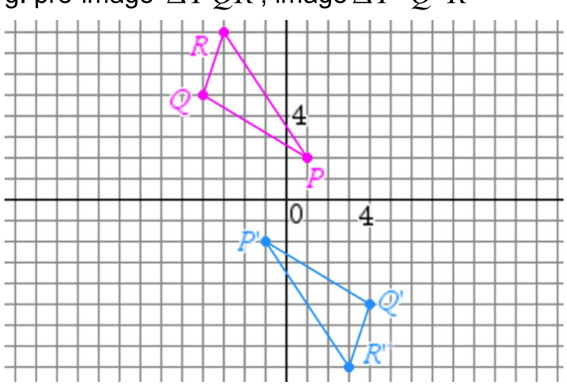
Translate ΔSTU 10 units right and down 4 units

f. pre-image ΔABC ; image $\Delta A'B'C'$



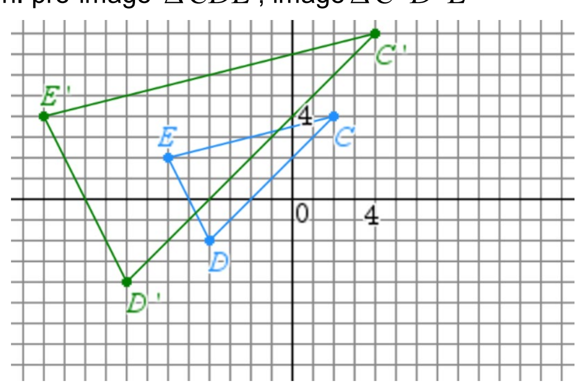
Reflect ΔABC about the x-axis.

g. pre-image ΔPQR ; image $\Delta P'Q'R'$



Rotate ΔPQR 180^o about the origin.

h. pre-image ΔCDE ; image $\Delta C'D'E'$



Dilate ΔCDE about the origin with a Scale Factor of 2.