Math Objectives

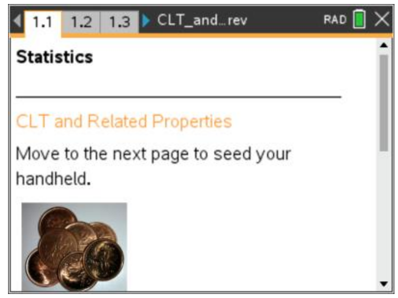
- Students will recognize that when *n* is sufficiently large, the sampling distribution of sample means, $\bar{x}$, is approximately normal, regardless of the shape of the population distribution (Central Limit Theorem).
- Students will recognize that when the population distribution is normal, the sampling distribution of sample means, $\bar{x}$, is normal for any sample size n.
- Students will recognize the consequences of the Central Limit Theorem when applied to quantitative data: a normal model with $m_{\bar{x}} = m$ (the true population mean) and $s_{\bar{x}}$ that decreases as sample size, *n*, increases.
- Students will recognize the consequences of the Central Limit Theorem when applied to proportions: a normal model with $m_{\hat{p}} = p$ (the true population proportion) and $s_{\hat{p}}$ that decreases as sample size, n, increases.
- Students will use appropriate tools strategically (CCSSM Mathematical Practices).

Vocabulary

- Central Limit Theorem (CLT)
- normal distribution
- population
- proportion
- quantitative data
- sample
- sample mean
- sample mean
- sampling proportion
- skewed right distribution
- spread
- uniform distribution

About the Lesson

- This lesson involves examining distributions of sample means of random samples of size *n* from four different populations.
- As a result, students will:
  - Observe a uniform distribution and click to see simulated sampling distributions of size *n*=1 to 30 with a normal curve imposed on the distribution in each case.
  - Consider the same questions with respect to a normal distribution, a skewed distribution and a proportion.
  - Observe that as the sample size gets larger, the better the simulated sampling distribution can be approximated by a normal model.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity
CLT and Related Properties_Student.pdf
CLT and Related Properties_Student.doc

TI-Nspire document
CLT and Related Properties.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to compare different random samples generated by students.
- Use Teacher Edition computer software to review student documents.

Use Quick Poll to assess students' understanding.

Discussion Points and Possible Answers

Tech Tip: Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a *Calculator* page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

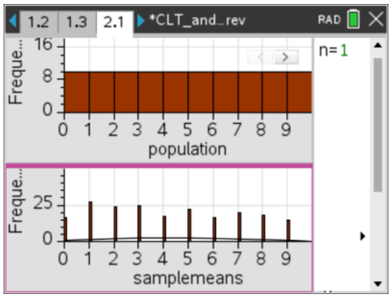
Teacher Tip: Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or the device is new, as otherwise the "random" numbers will all be the same as those on other similarly cleared devices.

Move to pages 1.2 and 1.3.

This activity involves generating a number of random samples from a population. In order to avoid having your results be identical to those for another student in the room, it is necessary to "seed" the random number generator. Read the instructions on page 1.2 for seeding your random number generator, then carry out that seeding on page 1.3.

Teacher Tip: Familiar hypothesis tests and confidence procedures for means of populations (*t*-procedures) or sample proportions (large sample *z*-procedures) assume that the population under question is normally distributed. It is usually impossible to check a real population, so it is necessary instead to examine the sample to see whether the population itself might reasonably be assumed to be normally distributed.

Move to page 2.1.



1. a. The graph on the top of Page 2.1 represents a population that is uniformly distributed—digits from zero to nine. Describe the shape of the population distribution.

   Sample Answers: The distribution is rectangular—all the bars are the same height.

   b. The lower half of the screen shows 200 random samples of size *n* = 1 from the population. Is this simulated sampling distribution similar to the population distribution? Explain.

   Sample Answers: Yes. The distribution is fairly uniform like the population.

   > **Teacher Tip:** Students might have trouble recognizing that the mean of a sample of size one is the sample value itself.

   c. Click on the right arrow to generate the means of 200 random samples of size *n* = 2 from the population. Describe how the shape and spread of the simulated sampling distribution changed and how it compares to the normal curve displayed with the simulated distribution.

   Sample Answers: With samples of size two (2), I can begin to see some mounding in the center of the distribution—it is no longer uniform in appearance. It isn't quite as wide as the original population, and the normal curve does not fit it at all.

   > **Teacher Tip:** The sampling distribution of sample means includes the means of all possible samples of a given size that could be chosen from the population. The simulations generate 200 samples of each size. Students will get different simulated sampling distributions with differing shapes and spreads, but the general pattern of approaching normality as sample size increases should be visible.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 1 at the end of this lesson.

   d. Click on the right arrow to generate the means of 200 random samples of size *n* = 5 from the population. Describe how the shape and spread of the simulated sampling distribution changed.

   Sample Answers: With samples of size five (5), the simulated sampling distribution is more mounded in the center with fewer values at the extremes. The distribution is narrower, and the normal curve fits a bit better.

2. a. Continue to click on the right arrow to generate 200 random samples of size n = 10, 15, 20, and 30 from the population. Describe how the shape and spread of the simulated sampling distributions changed as the size of the samples increased.

Note: Clicking on the left arrow will generate a new random sample of the next smaller size. You might want to click back and forth between sample sizes to better see the pattern.

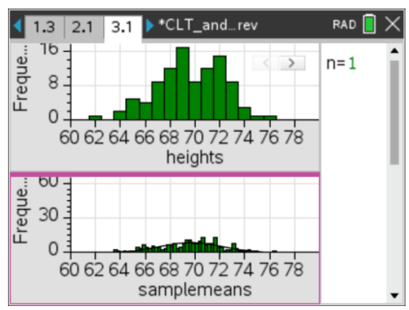
Sample Answers: As the sample size increased from 10 to 30, the simulated distributions generally got narrower and more mound-shaped. By sample size 30, the normal curve fits the distribution pretty well.

b. How does the simulated sampling distribution of sample means with samples of size 30 compare to the population distribution?

Sample Answers: The simulated sampling distribution of sample means with samples of size 30 no longer looks uniform like the original population. It is fairly mound-shaped and close to a normal distribution. The center of the sampling distribution appears to be about 4.5, the population mean. All of the sample means are between 3.5 and 5.5—a much narrower spread than the original population, which went from 0 to 9.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 1 at the end of this lesson.

Move to page 3.1.



3. a. The histogram on Page 3.1 represents the heights of young men. Their heights are normally distributed with a mean of 70 inches and a standard deviation of 2.5 inches. Describe the shape, center, and spread of the histogram of heights.

Sample Answers: The histogram is mound shaped and centered around 70 inches. The heights go from 62 to 76 inches.

b. The lower half of the screen shows 200 random samples of size n = 1 from the population. How does the simulated sampling distribution compare to the population distribution? Explain.

Sample Answers: The simulated sampling distribution for samples of size n=1 of heights is approximately mound-shaped and symmetric—similar to the population (approximately normal). The normal curve seems to fit the simulated distribution well.

c. Click on the right arrow to generate 200 random samples of size 2 and size 5 from the population of heights. Describe how the shape and the spread of the simulated sampling distributions change with these two sample sizes and how they compare to the normal curve displayed with the simulated distribution.

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Sample Answers: With samples of size of 2, the simulated distribution of sample mean heights stayed symmetric and got slightly narrower. With samples of size 5, the shape stayed about the same, and the distribution got even narrower. The normal curve still fits both fairly well.

4. a. Click on the right arrow to generate 200 random samples of increasing size (from 10 to 30) from the population of heights. Describe how the shape and the spread of the simulated sampling distributions change with increasing sample size and how they compare to the normal curve displayed with the simulated distribution.

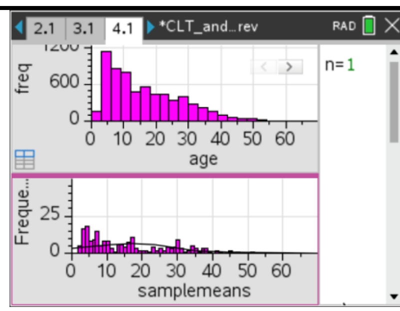
Sample Answers: As the sample size increased from 10 to 30, the simulated distributions of sample mean heights stayed mound-shaped and generally got narrower as sample size increased. The distributions got closer and closer to the normal curve, and for sample size 30, the curve was nearly a perfect fit.

b How does the simulated sampling distribution of sample mean heights with samples of size 30 compare to the population distribution of heights?

Sample Answers: The simulated sampling distribution of sample mean heights is mound-shaped and symmetric (or approximately normal) like the original distribution. The center of the simulated sampling distribution appears to be about 70, the given population mean. All of the mean heights for samples of size 30 are between 69.5 and 71 inches—a much narrower spread than the original population, which went from 61.5 to 76.5 inches.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

Move to page 4.1.



5. a. The histogram on Page 4.1 represents the ages of pennies collected by an AP Statistics teacher. Describe the shape, center, and spread of the histogram of penny ages.
Sample Answers: The histogram is highly skewed to the right (towards larger ages—older pennies). The tallest peak is at 3 to 6 years old, with the oldest pennies around 50-years old. I would guess the mean is around 15 years old.

b. The lower half of the screen shows 200 random samples of size *n* = 1 from the population. How does this simulated sampling distribution compare to the population distribution? Explain.

Sample Answers: The simulated distribution is still skewed right like the original population distribution.

c. Click on the right arrow to generate 200 random samples of sizes 2 and 5 from the population of penny ages. Describe how the shape and the spread of the simulated sampling distributions change with increasing sample size.

Sample Answers: For samples of size 2, the simulated sampling distribution was still somewhat skewed right but not as spread out. For samples of size 5, the simulated distribution had very little skew and was narrower. The normal curve was way off for both.

6. a. Click on the right arrow to generate 200 random samples of increasing size (from 10 to 30) from the population of penny ages. Describe how the shape, the center, and the spread of the simulated sampling distributions change with increasing sample size and how they compare to the normal curve displayed with the simulated distribution.

Sample Answers: As the sample size increased from 10 to 30, the simulated sampling distribution of sample mean penny ages became more mound-shaped and generally got narrower. The center appears to around 15 or 16. The normal curve fit better as the sample size increased.

> **Tech Tip:** The handheld (or software) is drawing 200 random samples from a population of over 6000, which takes some time. It might seem like the handheld is frozen.

b. How does the shape of the simulated sampling distribution of sample mean penny heights with samples of size 30 compare to the population distribution of penny ages?

Sample Answers: The population of penny ages was strongly skewed to the right. The simulated sampling distribution of sample mean penny ages for samples of size 30 was mound-shaped and symmetric (or approximately normal). The normal distribution fit very well for the sample of size 30.

c. How do the center and spread of the simulated sampling distribution of sample mean penny ages with samples of size 30 compare to the population distribution of penny ages?

Sample Answers: The center is about the same as I originally guessed for the population. All of the sample means are between 11.75 and 19.75 years—a much narrower spread than the original population, which went from 0 to over 50 years.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 1 at the end of this lesson.

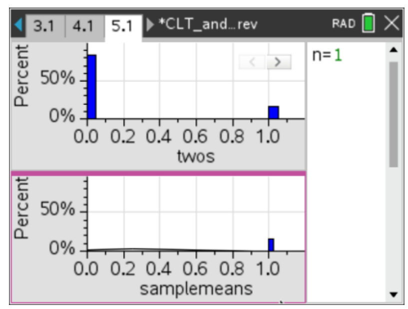
7. a. Imagine that you roll a die once and calculate the proportion of times a 2 landed face up. What are the possible proportions you can get?

Sample Answers: The possible proportions are 0/1 or 1/1.

b. In six rolls, about how often would you get a proportion of 0? of 1? What would the distribution of the proportions look like?

Sample Answers: I would expect to get five 0's (representing not rolling a 2) and one 1 (representing rolling a 2). The distribution would be bimodal.

Move to page 5.1.



c. The histogram on the top of Page 5.1 represents the proportion of dice rolls that are a 2 on a fair die. Describe the shape of the histogram of proportions. Does this match what you predicted in part b?

Sample Answers: The histogram is bimodal, with peaks at 0 and 1, which is what I predicted.

d. Click on the bar at 1. Explain what the percentage means in context. Does this match what you predicted in question 7b?

Tech Tip: Be careful not to drag the bar as you click on it.

Sample Answers: 16.7% means that the proportion of 2's rolled in the six rolls of the die is about 0.167 (16.7/100) or 1/6. Yes, this is what I expected to get.

e. The lower screen on Page 5.1 shows 200 random samples of size *n* = 1 from the population. How does this simulated sampling distribution compare to the population distribution? Explain.

Sample Answers: Yes. The distribution is still bimodal. The simulated sampling distribution with samples of size one should be the same shape as the original population—bimodal with peaks at 0 and 1.

f. Click on the right arrow to generate 200 random samples of increasing size (size 2 and size 5) from the population of dice rolls. Describe how the shape and the spread of the simulated sampling distributions of sample proportions change with increasing sample size.

Sample Answers: For samples of size 2, the simulated sampling distribution had three bars and was skewed to the right. For samples of size 5, the simulated distribution appeared to have four bars and was less skewed but is still far from normal.

g. For your samples of size 5, what does the bar over 0.2 represent?

Sample Answers: About 40% of the samples drawn have an average proportion between about 0.18 and 0.21, or about one 2 out of the five rolls.

8. a. Click on the right arrow to generate 200 random samples of increasing size (from 10 to 60) from the population of dice rolls. Describe how the shape, the center, and the spread of the simulated sampling distributions of sample proportions change with increasing sample size.

Sample Answers: As the sample size increased from 10 to 60, the simulated distribution of sample proportions became more mound-shaped and got narrower. The center appears to around 0.15, and the distribution appears to become more normal when compared to the graph.

b. How does the shape of the simulated sampling distribution of sample proportions with samples of size 60 compare to the population distribution of proportions?

Sample Answers: The population of proportions was bimodal. The simulated sampling distribution of sample proportions for samples of size 60 was somewhat mound-shaped and symmetric, and the normal curve fit the distribution fairly well.

c. How do the center and spread of the sampling distribution of sample proportions with samples of size 60 compare to the population distribution of proportions?

Sample Answers: The center is about the same as the proportion of rolls that would theoretically be a two—1/6. All of the sample proportions are between 0.05 and 0.25 while the population had values of 0 and 1.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 1 at the end of this lesson.

9. Describe what happens to the shape of the sampling distribution when you take large samples from a population (the Central Limit Theorem).

Sample Answers: The sampling distribution of sample means for large sample sizes will generally appear "roughly normal" when plotted. The shape of the original distribution does not matter—the shape of the original population distribution can have been normal, skewed, uniform, or bimodal.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The shape of the sampling distribution of sample means for a single sample varies from sample to sample, even when all the samples are selected randomly from the same population.
- Sampling distributions for samples with a large sample size will generally appear "roughly normal" when plotted, no matter the shape of the original population distribution (the Central Limit Theorem).
- The mean of the sampling distributions will be the population mean for all sample sizes. (The means of the *simulated* sampling distributions were *close* to rather than equal to the population mean for all sample sizes because students did not have the complete sampling distributions.)
- The variability in sampling distributions decreases as sample size increases.

Assessment

1. Which of the sampling distributions (uniform, normal, skewed, or proportions) will be approximately normal when the sample size is 1?

Answer: normal

2. A random sample is selected from a population of unknown shape with a mean $\mu = 25$ and standard deviation $\sigma = 5$. Fill in the chart below, stating whether the actual sampling distribution of $\bar{x}$ would always, sometimes or never be approximately normal for the given sample sizes. Also determine the mean of each sampling distribution of $\bar{x}$.

Sample size	Sampling distribution approximately normal?	Mean of the sampling distribution
n = 5		
n=10		
n=25		
n=60		
n=200		

Answer:

Sample size	Sampling distribution approximately normal?	Mean of the sampling distribution
n = 5	sometimes	25
n=10	sometimes	25
n=25	sometimes	25
n=60	always	25
n=200	always	25

Teacher Tip: Remind students that the sampling distribution of sample means includes *all* possible samples not just the 200 they simulated. If they had been able to generate the complete sampling distribution, the mean of the simulated sampling distribution would have equaled the population mean.

3. State the Central Limit Theorem.

Sample Answers. When *n* is large, the sampling distribution of the sample mean, $\bar{x}$, is approximately normal.

Teacher Tip: Remind students that the CLT only refers to the **shape** of the sampling distribution of sample means.

4 For proportion data, a sample size of 30 is always, sometimes, or never large enough for the distribution of sample proportions to be approximately normal? Explain your answer.

Sample Answers. Sometimes. Large enough for proportions requires np>10 and n(1-p)>10, so it depends on the proportion involved.

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Note 1

Question 2, 4, 6, 8 *Screen Capture*

Screen captures for questions 2, 4, 6 and 8 provide the opportunity to discuss simulated sampling distributions being approximately normal with large samples. Point out that no finite set can ever be exactly normally distributed. In addition, we don't have the entire sampling distribution—we only have 200 samples of the given size. Focus on the amount of variation from student to student (sample to sample) when it is known that all samples are of the same size and from the same population.