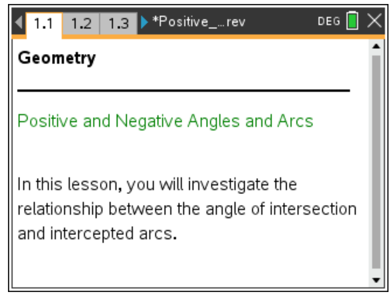
Positive and Negative Angles and Arcs
Student Activity

Name ________________
Class ________________

Open the TI-Nspire document *Positive_and_Negative_Angles_and_Arcs.tns.*

This activity begins with an interactive exploration with oriented angles. Then you will explore several cases in which two lines intersect at a point and also intersect a circle. You will investigate the relationship between the angle of intersection and the intercepted arcs.



An oriented angle is defined to be the measure of rotation of a ray about a common point (the vertex). One side of the angle is the initial ray and the other is the terminal ray, which is the result of a rotation of a given number of degrees, either in the clockwise or counterclockwise direction.

Move to page 1.2.

> Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

$\angle APB$ and $\angle BPA$ are the same angle but have different orientations. Because A is listed first in the labeling of $\angle APB$, $\overrightarrow{PA}$ is the initial ray and $\overrightarrow{PB}$ is the terminal ray. In the case of $\angle BPA$, $\overrightarrow{PB}$ is the initial ray and $\overrightarrow{PA}$ is the terminal ray.

If the initial ray is rotated in a counterclockwise direction toward the terminal ray, then its measure is positive. If the initial ray is rotated in the clockwise direction, then its measure is negative.

1. Drag point *A* counterclockwise past point *B*. What changes occurred in the angle measures?

Move to page 1.3.

2. The measure $\angle APB$ is shown as well as the measure of its reflex angle. A reflex angle is one that involves a rotation of more than $180°$, but less than $360°$.

   a. What do you notice about the relationship of the measures of these angles? How does it relate to the definition of an oriented angle?

   b. Drag point *A* counterclockwise along the circle past point *B*. What changes occurred in the angle measures? How do these changes relate to the definitions for an oriented angle and a reflex angle?

Move to page 1.4.

Both the minor arc *AB* and the major arc *ACB* are indicated on the circle. The measures of these arcs are also shown. Recall that the measure of an arc is equal to the measure of its central angle. The measure of $\angle AOB$ is also shown.

On this page, there is slider under the words *Show mArc:*. Notice that when the slider is set at *AB,* $\overrightarrow{OA}$ is the initial ray and the measures of both the minor arc *AB* and the major arc *ACB* are shown. Drag the point on the slider from *AB* to *BA*. Notice that $\overrightarrow{OB}$ is now the initial ray and the measures of minor arc *BA* and major arc *BCA* are shown.

3. Move point *A* around the circle with the slider in different positions. What changes do you see in the measures of these arcs? How do these changes relate to the definition of an oriented angle?

Move to page 2.1.

The measures of $\angle APB$, arcs *AB* and *CD* are shown. The arrows on arcs *AB* and *CD* indicate their orientation.

4. Drag point *P* around in the interior of the circle, but not at the center of the circle.
   a. With respect to the circle, what type of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

   b. What relationship do you notice between the measures of the arcs and the measure of $\angle APB$?

5. Drag point *P* to coincide with the center.
   a. With respect to the circle, what type of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

   b. Does the relationship you found in question 4b still hold? If the relationship you found does not hold, revise the relationship you stated so that it is true wherever *P* is in the interior.

6. Drag points *R* or *Q* so that the orientation of arc *CD* changes. Drag point *P* anywhere in the interior of the circle. Does the relationship you have found still hold? If not, revise the relationship so that it is true wherever *P* is in the interior of a circle and for both positive and negative arc measures.

7. Drag point *P* around the exterior of the circle, but make sure $\overleftrightarrow{PQ}$ and $\overleftrightarrow{PR}$ both intersect the circle in two points. Does the relationship you found still hold? If the relationship you found does not hold, revise the relationship so that it is true regardless of whether *P* lies in the interior or exterior of the circle.

8. Leave *P* in the exterior of the circle and $\overleftrightarrow{PR}$ intersecting the circle in two points. Drag point *Q* so that point *C* coincides with point *A*.
   a. With respect to the circle, what type of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

   b. Is the relationship you found still true? If the relationship you found is not true, revise it for this case.

9. Leave *P* in the exterior of the circle, and leave *C* so that it still coincides with point *A*. Drag point *R* so that point *B* coincides with point *D*.
   a. With respect to the circle, what type of lines are $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$?

   b. Let *CD* denote the major arc and *AB* the minor arc. Is the relationship you found still true? If the relationship you found is not true, revise it for this case.

10. Bryan says that if two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle. Which of questions 4–9, if any, support his conjecture?

11. Dajah says that if two secants intersect in the exterior of a circle, then the measure of the angle formed is one-half the sum of the measures of its intercepted arcs. Which of questions 4–9, if any, support her conjecture?

12. Michael agrees with Dajah, but he also thinks her statement is true for the case when there is a secant and a tangent, and also for the case when there are two tangents. Which of questions 4–9, if any, support his conjecture?

13. Chloe agrees with Bryan and Dajah, but believes that she has an idea that works for all of the cases. She thinks that if two lines intersect each other and also intersect a circle, then the measure of an angle of intersection of the two lines is the average of the angle's intercepted arcs. Which of questions 4–9, if any, support her conjecture?

14. Alejandro says the following formula works for questions 4–9. Do you agree or disagree? Why?

$$\angle APB = \frac{mArc\ AB + mArc\ CD}{2}$$