Teacher Notes



Analysis of a Bouncing Ball

Objectives

- Explore the relationships among position, velocity, and acceleration
- Connect mathematical relationships to real-world phenomena

Materials

- TI-84 Plus / TI-83 Plus
- Calculator-Based Ranger[™] (CBR[™])
- Ball (Racquetballs work well, as do basketballs or other balls that will bounce. Tennis balls may not give good results because the ball may absorb the sound wave rather than reflect it.)

Teaching Time

60 minutes

Abstract

In this activity, students will collect data for a ball bouncing under a CBRTM data collection device and select one bounce to analyze. They will fit an equation to the data with a quadratic regression and use the first and second derivatives to analyze the graphs of velocity and acceleration as a function of time. They will look for connections between the graphs and the physical motion of the ball.

Management Tips and Hints

Advance Preparation

This activity can also be done with a CBL2™ and Vernier motion detector. If using this equipment, load the BOUNCEIT program from the Real World Math with the CBL book or the Real World Math App from TI web site onto the students' graphing handhelds. The app can be found at **education.ti.com**. You may download the Real World Math with the CBL App which includes BOUNCEIT. The App can be found at **education.ti.com**. Search for Real World Math App. The application can be downloaded using the TI Connect™ software, also available at the same Web site. The Real World Math application has six separate downloadable Apps. The BOUNCEIT part of the App is located in CBLMath2. The BOUNCEIT program is also available on the CD or disk that comes with Real World Math with the CBL book, second edition.

Prerequisites

Students should:

- know how to take derivatives.
- be familiar with the numerical derivative key on the graphing handheld.

Student Engagement

Show the motion of the bouncing ball, and have students make predictions prior to the data collection. This may be done the day before the activity or as a homework assignment.

This activity is best done with students working in cooperative groups so that they can compare answers and look for connections. A class discussion of basic concepts after the activity would enhance understanding and ensure that students made correct observations.

Evidence of Learning

- Students are asked to make predictions prior to collecting data and graphing equations. This allows them to correct their own thinking.
- A class discussion or analysis of answered questions will demonstrate student knowledge.
- Students should realize that the velocity is positive when the ball moves upward and negative when the ball moves downward.
- Students should see that the velocity graph is linear for each bounce. The acceleration is always negative with a value of -9.8 m/s², which is the slope or rate of change for the velocity graph.

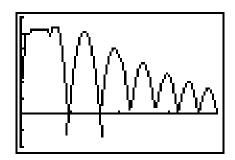
Common Student Errors/Misconceptions

- Many students have difficulties in interpreting velocity and acceleration graphs
 for a physical situation. They often believe that the acceleration of a ball at its
 maximum height is zero because the velocity is zero. Most students will not
 predict the velocity of a ball coming up from the ground to have a linear velocity
 equation with a slope equal to the acceleration because of gravity.
- Students do not need to bounce the ball high. Small bounces will generally help keep the ball under the CBR™. If the floor is carpeted or uneven, the ball and detector may be set up to bounce on a table. The ball does not need to bounce very high, and only one good bounce is needed for analysis.

Activity Solutions

- 1. n/a
- 2. n/a
- 3. n/a
- 4. Answers will vary depending on students' initial predictions.
- **5.** n/a
- **6.** $\frac{1}{2}g = a$; t = x; d = y
- **7.** The a value should be close to -4.9. The b and c values will vary.
- **8.** The value of g should be close to -9.8 m/s². The value of a is $\frac{1}{2}g$.

- **9.** The values should be close.
- **10.** The values of b and c would represent the initial position and initial velocity if the initial time for the selected parabola were t = 0. But because we selected a portion of the time and height data and the starting time for that parabola is not zero, the values do not have physical significance. The diagram shows the parabola and equation for the data. The values of b and c can be likened to the initial velocity and initial position of the ball if it were launched from a hole in the ground at time t = 0.

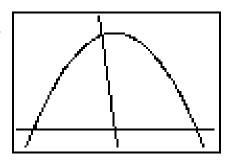


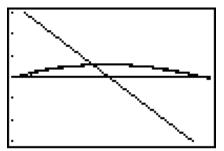
The parabola could be shifted to the left by subtracting the time where the regression equation intersects the time axis from all values in list $\mathbf{L5}$. If the quadratic regression were performed on these new time values, the b and c values would be approximately the initial velocity and initial position.

- **11.** Answers will vary.
- **12.** The values should be positive on the left, nearly zero at the top and negative on the right side of the parabola.
- **13.** Answers will vary. The graph should begin with a positive velocity value and decrease to negative values.

14. n/a

15. Answers will vary. Two sample graphs are shown. Students' graphs should include viewing window settings for the minimum and maximum *y*-values.





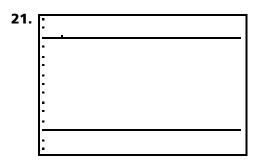
16. Answers will vary.

- **17.** The velocity is positive when the ball is moving upward, zero at the top, and negative as the ball moves back toward the ground.
- 18. Acceleration is the instantaneous rate of change of velocity, or acceleration is

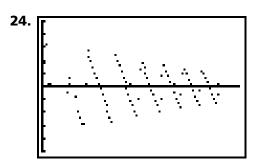
$$\frac{\Delta v}{\Delta t}$$

as Δt approaches zero, or acceleration describes how the velocity changes each second.

- **19.** The slope of the velocity versus time graph is constant with a value of approximately -9.8 m/s because the velocity is decreasing by 9.8 m/s each second. The slope of the velocity graph is never zero.
- **20.** The prediction should be a horizontal line at approximately -10.



- **22.** The acceleration is constant during the bounce.
- **23.** The velocity versus time plots would be a series of parallel lines because the acceleration is constant.



25. Answers will vary.