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| **Math Objectives**   * Students will graph linear functions and determine properties such as slope and y-intercept, describe how both change as the graph is rotated and translated, and understand that the coordinates of a line are the solution set to the equation of the line. * Students will show that lines with equal slopes are parallel and slopes that are negative reciprocals are perpendicular. * Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.   **Vocabulary**   * Parallel • Perpendicular • Reciprocal * Slope   **About the Lesson**   * This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL * This falls under the IB Mathematics Core Content Topic 2 Functions:   **2.1**: **(a)** Different forms of the equation of a straight line  **(b)** Gradient; intercepts  **(c)** Lines with gradients *m1* and *m2*  **(d)** Parallel lines *m1 = m2*  **(e)** Perpendicular lines *m1 x m2 = -1*  As a result, students will:   * Apply this information to real world situations.   **HH_SW_iconsTI-Nspire™ Navigator™**   * Transfer a File. * Use Class Capture to examine patterns that emerge. * Use Live Presenter to demonstrate. * Use Teacher Edition computer software to review student documents. * Use Quick Poll to assess students’ understanding.-   **Activity Materials**  Compatible TI Technologies: **Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Handheld_icon.png** TI-Nspire™ CX Handhelds,  Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Tablet_icon.png TI-Nspire™ Apps for iPad®, Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Software_icon.png TI-Nspire™ Software   |  | | --- | | **Tech Tip:** Before beginning the activity, the file *PointsLinesSlopesOhMy.tns* needs to be transferred to the students’ calculators through the navigator or computer to handheld transfer. | | **Tech Tips:**   * This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld. * Watch for additional Tech Tips throughout the activity for the specific technology you are using. * Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>   **Lesson Files:**  *Student Activity*  PointsLinesSlopesOhMy\_Student-Nspire.pdf  PointsLinesSlopesOhMy\_Student-Nspire.doc  PointsLinesSlopesOhMy.tns | |
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The right, left, up and down directional arrows on the touch pad will move the point on grid points. Students might also drag the point. Check to make sure that they have moved the cursor (arrow) until it becomes a hand getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press **ctrl** and the **center of the touchpad** to grab the point and close the hand. When finished moving the point, press **esc** to release the point. |   **Open the TI-Nspire document *Points\_on\_a\_Line.tns.***  **Move to page 1.2**  **Problem 1 – Coordinates of Points**  First, a little review on coordinate location within the four quadrants. Place point *P* in the top right quadrant (**menu, Geometry, Points & Lines, Point**). Drag the point around into different quadrants. Complete the sentences by writing *positive* or *negative*.  1. A point is in Quadrant 1 (top right) when its *x*-coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and its *y*‑coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_. **Solution:** positive and positive  2. A point is in Quadrant 2 (top left) when its *x*-coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and its *y*‑coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_. **Solution:** negative and positive  3. A point is in Quadrant 3 (bottom left) when its *x*-coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and its *y*‑coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_. **Solution:** negative and negative  4**.** A point is in Quadrant 4 (bottom right) when its *x*-coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ and its *y*‑coordinate is \_\_\_\_\_\_\_\_\_\_\_\_\_\_. **Solution:** positive and negative   |  | | --- | | **Teacher Tip:** Students should now be able to make sense of exactly what the coordinates should be in different quadrants as well as on each axis. Give students the coordinates of a point and ask which quadrant it is in and visa versa. This simple activity is good for students to develop their technology skills and establish sign patterns for the four quadrants. |   **Move to page 1.3.**  **Problem 2 – Slope Review**  Suppose you wanted to go from A to B, but you could only make a vertical move up or down or a horizontal move right or left. Notice there is an arrow for the vertical trip from A to C and an arrow for the horizontal trip from C to B.  5. Describe the vertical and horizontal moves you would make to get from point A to point B.  **Solution:** You would move up 7 units and to the right 14 units.   |  | | --- | | **TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)***  Consider introducing the file using Live Presenter. To clarify the difference between horizontal and vertical moves, ask the student presenter to identify the vertical and horizontal legs of the triangle |   6. a. Move point A until the vertical path from point A to point C is up 2 spaces. Describe  the horizontal move to get to point B from point C.  **Solution:** The horizontal move would be 4 units to the right.  b. Move point A until the vertical path is up 3 spaces. Describe the horizontal move to  get to point B from point C.  **Solution:** To move from *C* to *B*, you have to move to the right 6 units.  c. Write down the horizontal move that will correspond to a vertical move of up 6. Move  the point to check your answer.  **Solution:** To move from *C* to *B*, you have to move to the right 12 units.     |  | | --- | | **Teacher Tip:** Students should realize that when moving from one point to another, the vertical and horizontal moves involve direction as well as distance. Be sure studetns remember that the quadrants are labeled from 1 to 4 counterclockwise – beginning with the first quadrant where x and y are both positive. And be sure that the students are describing both the distance and the direction for this problem. |   7. Make an educated guess about the relationship between the number of units and the direction from A to C and from C to B. Choose some new points for A and B, and verify your conjecture.  **Possible Answers:** The number of units vertically and horizontally from *A* to *C* is half the   number of units from *C* to *B* (or the number of units from *C* to *B* is double the number of units   from *A* to *C*). Students might note that if the direction from *A* to *C* is up, then the direction from   *C* to *B* is right. If the direction from *A* to *C* is down, then the direction from *C* to *B* is left.   |  | | --- | | **Teacher Tip:** If students move the points so they coincide, **ctrl Z** will undo the move so the two points are once again distinct. |   8. Describe what happens when point A is above and to the right of point B. Try several points for which this is true. Explain if the results support your conjecture.  **Possible Answers:** The vertical change is down, and the horizontal change is to  the left. The student could count and observe the direction of the arrows.  In a coordinate system, a move up is considered a positive vertical change; a move down is a negative vertical change; a move right is considered a positive horizontal change; a move left is a negative horizontal change.  9. Using correct signs, find the ratio of vertical change to horizontal change for several pairs of points on the line. Explain what you observe about the ratios.  **Possible Answer:** The ratios are equivalent and are all ½ .  **Move to page 1.4.**  10. Move points A and B to fill in the missing information in each line of the table below. Explain your reasoning.  **Solution:** See table below   |  |  |  |  | | --- | --- | --- | --- | |  | Coordinates of Point *P* | Coordinates of Point *Q* | Vertical Change  Horizontal Change | | 1 | ( -8 , **-2** ) | ( **6** , 5 ) |  | | 2 | ( -6 , **-1** ) | ( **-2** , **1** ) |  | | 3 | ( **2** , 3 ) | ( **8** , **6** ) |  | | 4 | ( 6 , **5** ) | ( **-6** , **-1** ) |  |  |  | | --- | | **Teacher Tip:** Students might struggle with the fact that a ratio with two negative signs is the same as the ratio where both numerator and denominator are positive. Some of them might also be subtracting the values to find the vertical and horizontal changes, but depending on the order in which they subtract, they might get both positive or negative values. Be sure to stress that if they do this, they have to start from the same point to calculate both the horizontal and vertical change. |   11. Describe how the information in the table in question 10 relates to your observations in question 9.  **Answer:** The ratios are equivalent and are all ½ .  12. Suppose points A and B are on the line but not displayed in the window of the document. If the vertical change from point A to point B is 50, find the horizontal change. Explain your reasoning.  **Possible Solution:** The horizontal change would be 100. The ratios has to  remain ½ , so if the numerator is 50, the denominator has to be 100.   |  | | --- | | **Teacher Tip:** The vertical change of 50 means “up 50” or “positive 50.” Ask the students to consider if the vertical change is “negative 50” or if the horizontal change is 50. |   13. For a different line, the coordinates of point A are (-3, -4), and the ratio of the vertical change to the horizontal change is equivalent to . Find the coordinates of another point on the line. Explain how you found your answer.  **Possible Solutions:** Many different responses are possible. (0, -2), (3, 0), (6, 2). “I  moved up 2 and right 3.”   |  | | --- | | **Teacher Tip:** Students should be able to show how their points are generated using the ratio 2/3. Sharing responses will help them realize that the ratio of the vertical change to the horizontal change for any two points on a line is constant. You might want to use page 2.1 of the *tns* file. This page allows points *A* and *B* to be freely moved on the grid so that many different lines can be displayed. |   **Move to page 2.1.**  **Problem 3 – Lines, Equations, and Slopes**  Look for relationships between the points, slope, and equation as you change the line by grabbing and dragging point A, and then by grabbing and dragging the line itself.  14. Place a point on the line and label its coordinates. Drag the point along the line and record several coordinates of points. Explain how the coordinates relate to the equation of the line.  **Solution:** Coordinates will vary. Each ordered pair is a solution to the equation of the line.  15. When dragging the line by point A, describe the relationship between the points and the slope.  **Solution:** The slope is the change in the y-values divided by the change in the x-values.  16. When dragging the line by a point, describe the relationship of the slope and the equation.  **Solution:** The slope changes. The coefficient in front of the variable x changes.  17. To the right is a graph with two points labeled. Consider the line through these points. Then, consider the graph of the equation . Show you work and explain how the two lines compare. Especially consider the slope and *y*-intercepts.    **Solution:** The slope of the graph is (1 – (–3))/(3 – 1) = 4/2 = 2. This is a larger slope than the   equation which has a slope of ½. The graph is much steeper than the equation. The *y*-intercept  of the equation is positive 2, whereas the *y*-intercept of the graph is a negative number. From  the equation of the graph, *y* + 3 = 2(*x* – 1) or *y* = 2*x* – 5, it can be seen the *y*-intercept is –5.  **Move to page 2.2.**  **Problem 4 – Slopes of Parallel and Perpendicular Lines**  Drag the lines by points A and B and examine the slopes.  18. Explain what you notice about the slopes of two parallel lines.  **Solution:** The slopes of the two parallel lines are equal.  **Move to page 2.3.**  Drag the lines by points A and B and examine the slopes.  19. Explain what you notice about the slopes of two perpendicular lines.  **Solution:** Students will observe that when one slope is positive, the other slope is negative.  They may need to continue exploring to discover that the slopes of the perpendicular lines are   the opposite reciprocals of each other.  Discuss with a classmate what happens when the slopes of two perpendicular lines are multiplied together. Test your theory on the current slopes of both lines. Now, change the lines by grabbing and dragging point A and test your theory several times*.*  20. Describe what you observe about the product of the slopes.  **Solution:** The product is always -1. The slope of perpendicular lines is the opposite reciprocal   of each other.  **Further IB Application**  Hannah has always liked the kite shape. She plans to tile her bathroom floor with a pattern made up of kites. For the patter, she will be designing her initial kite *PQRS* on a set of coordinate axes in which one unit represents 5 cm.  The coordinates of *P, Q,* and *R* are (2, 0), (0, 4), and (4, 6) respectively. Point *S* lies on the x-axis. PR is perpendicular to QS. See the diagram below.  Diagram not to scale.  (a) Find the gradient of PR.  **Solution:**  (b) Write down the gradient of QS.  **Solution:**  (c) Find the equation of QS and write the answer in the following forms:  (i) y = mx + c    **Solution:**  (ii) y – y1 = m(x – x1)  **Solution:**  (iii) ax + by + d = 0  **Solution:**  (d) Find the coordinates of point *S*.  **Solution:** | | |
| |  | | --- | | **TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)***  **Any part to any Problem in the activity would be a great way to quickly assess your student’s understanding of linear functions and their slopes, along with using any student to be the presenter throughout the activity.** |  |  | | --- | | **Teacher Tip:** Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review linear functions and their characteristics, but also to generate discussion. |   *\*\*Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.* | | |