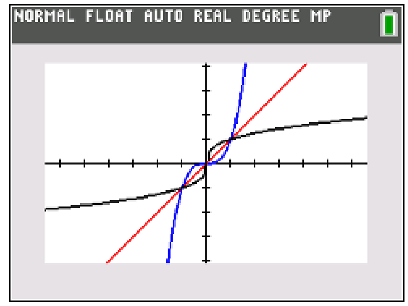
Inverse Functions

IB Aligned



Math Objectives

- Students will discover and discuss the graphical and algebraic relationships a function has with its inverse and the line $y = x$.
- Students will apply inverse functions to real world situations including temperature and money conversions.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Inverse
- Reflection
- Domain
- Range

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 4 Functions:
  2.2e Inverse function as a reflection in the line $y = x$ and the notation $f^{-1}(x)$.
  2.5c (AA only) Finding the inverse function $f^{-1}(x)$.
  2.14b (AA HL only) Finding the inverse function $f^{-1}(x)$, including domain restrictions.
- As a result, students will:
  - Apply this information to real world situations

Teacher Preparation and Notes

- This activity is done with the use of the TI-84 family as an aid to the problems.

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Activity Materials

- Compatible TI Technologies:
  TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint^{TM} functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Inverse_Functions_Student-84.pdf
Inverse_Functions_Student-84.doc

The prior knowledge needed for this activity is the relationship between a function and its inverse. The first few problems will revisit these relationships before asking you to apply this knowledge to real world scenarios.

First, we will review the graphical relationship. Graph the function $f(x) = -1 + \sqrt{x-1}$ and the line $y = x$ into Y1 and Y2 respectively.

> **Teacher Tip:** Although the topic of inverse functions and their algebraic and graphical relationships is learned early on in Algebra 1, Algebra 2, and Precalculus, many students forget about these relationships and how to apply them to different problems. Make sure you are circling the classroom as they discuss each of these problems to ensure they are making the connections.

Problem 1

Take a moment to discuss with a partner the significance of the line $y = x$ with respect to the function and its inverse. Share your thoughts with the class.

Possible discussion points: A function's inverse is a reflection in the line $y = x$. To reflect over the line $y = x$, you would transform a coordinate (x, y) onto (y, x).

Problem 2

Now let us discuss an algebraic relationship a function has with its inverse, finding a function's inverse. We use a two-step process. With the given function, you will first switch the x and y, and second, you will solve for y:

Given Function:

$y = -1 + \sqrt{x-1}$

Switch x and y:

$x = -1 + \sqrt{y-1}$

Solve for y:

$x + 1 = \sqrt{y-1}$

$(x+1)^2 = y - 1$

$y = f^{-1}(x) = (x+1)^2 + 1$

**<u>Further practice:</u>

Find the inverse of each function.

(a) $f(x) = 3x - 7$

Solution: $f^{-1}(x) = \frac{x+7}{3} = \frac{1}{3}x + \frac{7}{3}$

(b) $f(x) = \sqrt[3]{x+5} - 2$

Solution: $f^{-1}(x) = (x+2)^3 - 5$

(c) $f(x) = 2 + \frac{5}{x-4}$

Solution: $f^{-1}(x) = \frac{5}{x-2} + 4 = \frac{4x-3}{x-2}$

Problem 3

Graph the example demonstrated from **Problem 2** into Y3. Discuss with a partner what you notice after graphing the new function. Share your thoughts with the class.

Possible discussion points: The inverse function graphed was a complete parabola whereas the original function was a square root function and only half a parabola. This could lead to a discussion of restricted domains and how the domain and range are affected by the reflection in the line $y = x$.

Problem 4

Extension Question:
What is the relationship between the domain and the range of a function and its inverse?
Use the graphical and algebraic relationships on the previous pages to discuss this with a partner.
Share your results with the class.

Possible discussion points: A reflection in the line $y = x$ requires one to switch x and y, which is the first step in finding a function's inverse. This also leads to a function's domain equating to its inverse's range and a function's range equating to its inverse's domain.

> **Teacher Extension:** If time permits, this is a great place to have students compare values in the table. They can easily see how the x and y values are just switched with the side by side comparison.

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review inverse functions, but also to generate discussion.

Extension

Problem 5

Real World Inverse Function Applications Example 1:

Temperature Conversions
(°F → °C and °C → °F)

The formula to convert temperatures from degrees Celsius to Fahrenheit is $°\mathrm{F} = \frac{9}{5} \cdot °\mathrm{C} + 32$.

(a) Write the inverse function, which converts temperatures from Fahrenheit to Celsius.

Solution: $°\mathrm{C} = \frac{5}{9}(°\mathrm{F} - 32)$

(b) Find the Celsius temperature that is equal to 89 degrees Fahrenheit.

Solution: 31.7 ℃

(c) Explain how you could have found the answer to part (b) without finding the inverse function.

Possible discussion points: Using the idea that a function's domain is equal to the inverse's range and using the table feature on the Nspire to find the corresponding temperature to approximately 89 °F.

Problem 6

Real World Inverse Function Applications Example 2:

Money Conversions

A Canadian traveler who is heading to the United States exchanges some Canadian dollars for U.S. dollars. At the time of his travel, $1 Can = $0.79 U.S.
At the same time an American business woman who is in Canada is exchanging some U.S. dollars for Canadian dollars at the same exchange rate.

(a) Write an equation that gives the amount of money in U.S. dollars, d, as a function of the Canadian dollar amount, c, being exchanged.

Solution: $d = 0.79c$

(b) Find the amount of money in U.S dollars that the Canadian traveler would get if he exchanged $500.

Solution: $d = 0.79c = 0.79(500) = \$395\ U.S.$

(c) Find the amount of money in Canadian dollars that the American Business woman would get if she exchanged $1000 U.S.

Solution: $c = \frac{d}{0.79} = \frac{1000}{0.79} = \$1{,}265.82\ CAN$

(d) Explain why it might be helpful to write the inverse of the function you wrote in part (b) to answer part (c). Then, write an equation that defines the inverse function.

Possible solution: Since you are reversing the exchange process, it will be helpful to undo the operation of the initial money exchange. You will change from multiplying by 0.79 to convert to US dollars to dividing by 0.79 to convert to Can dollars. The equation would be $c = \frac{d}{0.79}$.

Further IB Style Question:

The price of a liter of soda at Carl's Convenient Store is $1.20. Carl's is having a sale on soda. If you purchase a minimum of 8 liters, a $4 discount is applied to your total. This can be modeled by the function, S, which gives the total cost when buying a minimum of 8 liters of soda.

$$S(x) = 1.20x - 4, \qquad x \geq 8$$

(a) Find the total cost of buying 10 liters of soda at Carl's. [2 marks]

Solution: $S(10) = 1.20(10) - 4 = \$8$

(b) Find $S^{-1}(26)$. [2 marks]

Solution: Knowing that the range of function is equal to the domain of the inverse, then

$$26 = 1.20x - 4$$
$$x = 25\ liters$$

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