Too Many Choices!

ID: 11763

Time required
40 minutes

Activity Overview

In this activity, students will investigate the fundamental counting principle, permutations, and combinations. They will find the pattern in each situation and apply it to make predictions. After a teacher-led discussion on the formulas, students will apply them to several problems.

Topic: Probability

- *Counting methods*
- *Permutations*
- *Combinations*

Teacher Preparation and Notes

- *This activity should be teacher-led. It allows for some student discovery and some inquiry questioning by the teacher.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11763" in the keyword search box.***

Associated Materials

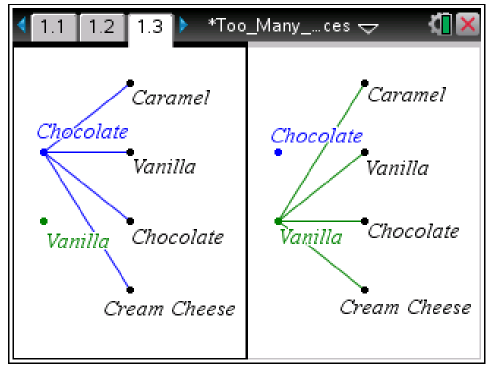
- *Too_Many_Choices_Student.doc*
- *Too_Many_Choices.tns*
- *Too_Many_Choices_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

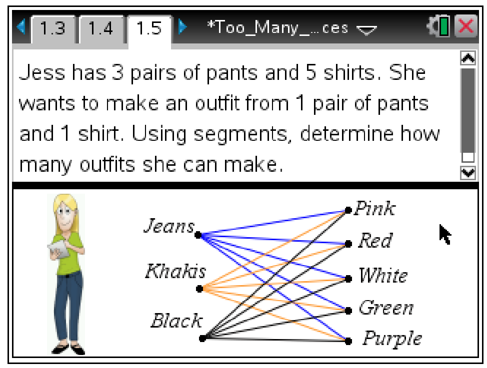
- *Combinations (TI-Nspire technology) — 8433*
- *Permutations (TI-Nspire technology) — 8432*
- *How Likely Is It? Exploring Probability (TI-Nspire technology) — 9236*

Problem 1 – Exploring the Fundamental Counting Principle



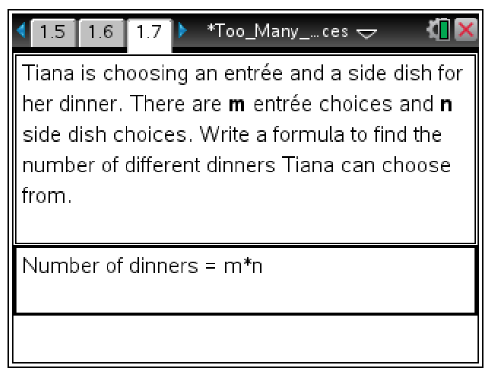
On page 1.3, students are shown how to use segments to determine the different cakes that Jayden will be able to choose from. For clarity, the frosting choices for Chocolate cake are on the left and the frosting choices for Vanilla cake are on the right.

Students are to determine a multiplication sentence that represents the problem. This will help them develop a formula at the end of this part of the activity.



On page 1.5, students will use the **Segment** tool (**MENU > Points & Lines > Segment**) to help them determine the number of outfits Jess has to choose from. Then they can create the multiplication sentence, pairs of pants × shirts.

If students need to they can change the attributes of the segments (**MENU > Actions > Attributes**) so that the segments for Jeans, Khakis, and Black are different in appearance.



Students are given a general problem of **m** entrees and **n** sides. They need to determine the formula to find the total number of meals Tiana can choose from (**m** × **n**).

Discuss with students the **Fundamental Counting Principle**, which says that if one event can happen **m** ways and a second event can happen **n** ways, then together they can happen **m x n** ways.

Then students can work through Try These on the worksheet using the Counting Principle.

1. 31 · 30 = 930 days 2. 10 · 10 · 10 · 26 · 26 = 676,000 plates

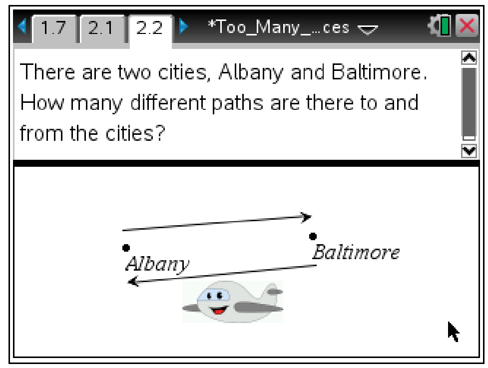
TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

Discussion Questions

- How would the number of ice cream cones change if the parameters were changed? (The two flavors could be the same. Strawberry/Vanilla and Vanilla/Strawberry are considered the same cone.)
  - Would the number of cones be more or less?
  - What mathematical operation must take place for this to happen?
- How would the number of license plates change if the digits could not be repeated?

Problem 2 – Exploring Permutations



Students are to investigate the number of paths that connect two cities for a given number of cities on pages 2.2–2.5. It is important to know that a trip from Albany to Baltimore is not the same as a trip from Baltimore to Albany. They will use the answers from those pages to fill in the first four rows of the chart.

Note: It is important for students to understand that they are finding the number of paths from one city to another no matter how many total cities are on the page.

When there are 3 cities the paths are:

Chicago to Albany	Albany to Chicago
Albany to Baltimore	Baltimore to Albany
Baltimore to Chicago	Chicago to Baltimore

Number of Cities	Number of Paths
2	2
3	6
4	12
5	20
6	30
7	42

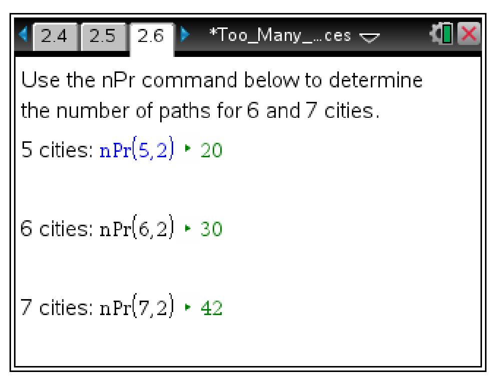
Encourage students to look for a pattern and determine a formula they think will find the number of paths for *n* cities. They can use that formula to calculate the paths for 6 and 7 cities, completing the last two rows.

Discuss with students the definition of permutations and the formula to compute the answer.

Explain that this arrangement of the paths is an example of a **permutation**, an arrangement of objects in which order matters. In general, this can be written:

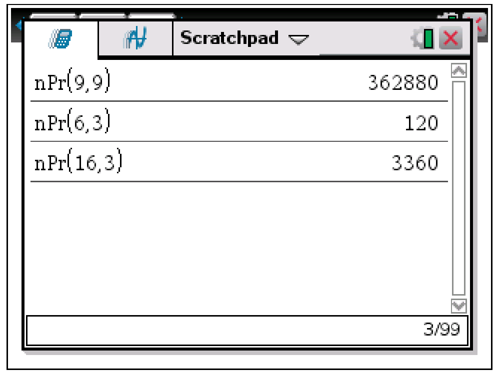
$${}_nP_r = \underbrace{n(n-1)(n-2)\dots}_{r\text{ factors}},$$

where *n* is the total number of objects and *r* is the number to be arranged.



Then, students are to complete the Try These problems using the ***nPr*** command and the Scratchpad .

1. nPr(9,9) = 362,880 batting orders
2. nPr(6,3) = 120 slates of officers
3. nPr(16,3) = 3360 ways



Discussion Questions

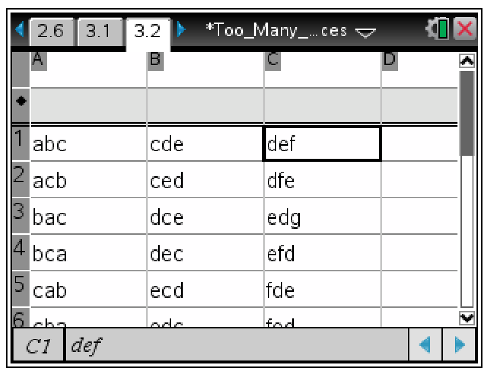
- How does the formula for permutations follow from the fundamental counting principle?
- Introduce the factorial notation (n!)

Problem 3 – Exploring Combinations

Students will now focus on a version of <u>Try These</u> problem 2 to consider a committee of three versus a slate of officers. Page 3.2 illustrates the number of slates that correspond to one committee. Column A lists the possible combinations of a committee with people a, b, and c. Column B lists the possible combinations of a committee with people c, d, and e.

Make sure that students understand that order does not matter for a committee. So, the combinations in a column represent one committee.

Students are encouraged to find another set of slates that correspond to one committee of people d, e, and f. They should see that there are still 6 possible combinations. For further investigation, have students determine how many slates there would be for a committee of 4 or 5, etc.



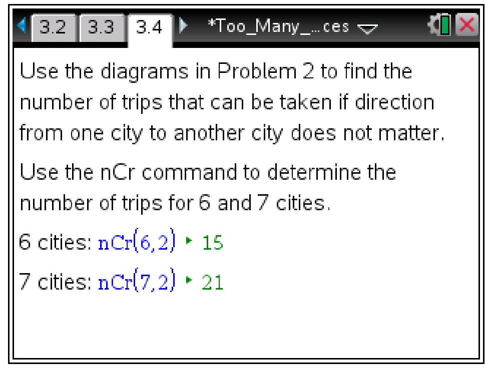
Students then continue their investigation of the paths from Problem 2, but now the trips where Albany to Baltimore is the same as Baltimore to Albany. They should see that the number of trips is half the number of paths.

Discuss with students the definition of combinations and the formula to compute the answer.

Number of Cities	Number of Paths	Number of Trips
2	2	1
3	6	3
4	12	6
5	20	10
6	30	15
7	42	21

The two problems are examples of **combinations,** an arrangement of objects in which order does not matter. In general, this can be written:

$$ {}_nC_r = \frac{\text{number of permutations}}{r!} $$

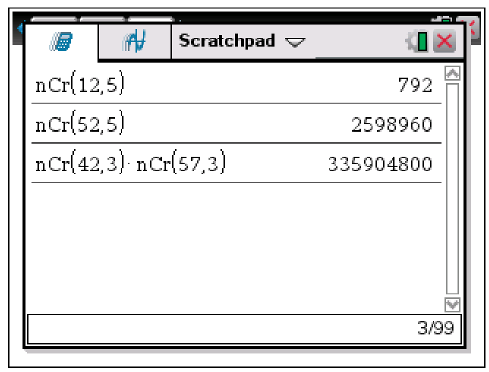


Discussion Questions

- Why does the formula take the number of permutations and divide by ***r*!**?
- Can there ever be more combinations than permutations for the same number of elements?
- Can ${}_nP_r = {}_nC_r$ for the same n and r?

Students are to use the combinations formula and the **nCr** command to answer the Try These problems. These calculations can be done on the Scratchpad .

1. nCr(12,5) = 792 teams
2. nCr(52,5) = 2,598,960 hands
3. nCr(42,3)*nCr(57,3) = 335,904,800 committees



TI-Nspire Navigator Opportunities

Note 1
Problems 1–3, *Quick Poll*

You may choose to use *Quick Poll* to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.

